# A contribution to the first Hilbert problem

Basis of our investigations is a countable order of everything thinkable. On this basis it will be shown that all proofs of the existence of uncountable sets contains a contradiction. As a concrete example, the second diagonal argument of Cantor will be quoted and a contradiction in this argument will be proved. This simultaneously solves the first-Hilbert problem.

We first examine all possible persons P, which read at any possible time point T any information in the form of a written Message M. If such a person P at a time point T is willing to say, the message M says "something" clearly and consistently, we call this something" object of thought of P" and name it "OT(P, T, M)". So the author would be willing to say, at a time point when he writes this paper, the message M = "2" describes the natural number two clearly and consistently or the message M = "i" describes the letter i clearly and consistently . In another context he would be willing to say, the message M = "i" describes the number $\sqrt{-1}$. Depending on the object of thought is then OT(P,T,M) = 2 or OT(P,T,M) = i or OT(P,T,M) = $\sqrt{-1}$.

To get to the desired countable order of everything thinkable, we introduce one after the other countable arrangements for all possible persons P, all possible time points T and all possible messages M

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Any possible person P must take at a time point T while reading the message M a certain minimum volume in space. The process of reading requires a certain minimum time. It can be assumed both is extensive enough to include at least one elementary cube EC(P,T) in the space-time universe entirely, if the side length of the cube is fixed at 0.01 mm and its duration at 0.01 seconds. Now we introduce a four-dimensional coordinate system in the space-time universe. In this coordinate system apparently all sorts of elementary cubes EC(P,T) can be arranged countable. We name this countable arrangement AR[EC(P,T)] = AR(P,T). Every possible process of reading of a person P at a time point T has a permanent place in this countable arrangement.

Next, we arrange countable all sorts of messages. Without loss of generality, we restrict ourselves to written Messages. A "Message of size n" should be a square grid consisting of $n^2$ "elementary squares" of side length 1/100 mm, each of which is either black or white, arranged in n rows of n places. To a white elementary square we assign the number 1, to a black one the number 2. The elementary square that stands in line j at the place k we denote by $a_{jk}$. Any possible message M of size n is then clearly represented by the decimal number $a(M) = 0,a_{11}...a_{1n}a_{21}...a_{2n}...a_{jk}...a_{nn}$. Now we sum up all the messages first in groups according to their size n, arrange them within each group according to the size of a(M) and arrange them finally in an countable arrangement AR(M).

All possible objects of thought OT(P,T,M) can now as requested, using the arrangement AR (P,T) be arranged into groups and then each with the help of the arrangement AR(M) in a countable arrangement AR[OT(P,T,M)].

As an example, we consider RN(0.1), the set of real numbers between 0 and 1. We will show that the second diagonal argument of Cantor used as a proof for the uncountability of RN(0.1) contains a contradiction. For this purpose we start from the countable arrangement AR[OT(P,T,M)], and select those objects of thought, for which P at time point T says, M describes for him a real number between 0 and 1 clearly and consistently. These selected objects of thought we call OT{P[RN(0.1)],T[RN(0.1)],M[RN(0,1)]}. As a part of the countable arrangement AR[OT(P,T,M)] they can also be arranged countable and we name this countable arrangement ARRN (P,T,M).

We now claim all real numbers between 0 and 1 are contained in the countable arrangement ARRN(P,T,M). A critic of our argument, let's call him PC, wants to prove the incompleteness of ARRN(P,T M) using the second diagonal argument of Cantor. For this he brings each real number $r_n$ aus ARRN(P,T,M) in the form of an infinite decimal $r_n = 0,r_{n1}r_{n2}---r_{nn}...$ and forms a diagonal number $d = 0,d_1d_2...d_n...$ with the property $\forall k$: $d_k \neq r_{kk}$. The critics argue that the diagonal number d is obviously a real number between 0 and 1, but it differs in each case in the $k^{th}$ decimal place from $r_k$. It is therefore $\forall k$: $d \neq r_k$ and therefore **d ∉ ARRN(P,T M).** Therefore the arrangement ARRN(P,T,M) does not contain all real numbers between 0 and 1.

PC obviously is able to bring its description of the diagonal number d in the form of a written Message, we call it MC. Is TC a time point in which he expresses his criticism, the object of thought OT(PC,TC,MC) describes due to his own statement the real number d between 0 and 1 clearly and consistently. It is therefore not only d ∈ AR[OT(P,T,M)] by definition of AR[OT(P,T,M)] but also **d ∈ ARRN(P,T M)** by definition of ARRN(P,T,M). Thus the required contradiction has been demonstrated.

The error of the critic is based on the fact, that ARRN(P,T,M) is only potentially fully available. Actually are missing **always** infinitely many real numbers. The application of the second diagonal argument to AORZ (P, T, M) leads to a circular argument. It can only work if there is an incomplete order. Only then it leads to a new real number between 0 and 1. The incompleteness the critic wants to prove is therefore assumed implicitly from the start.