A universal ordering

1. Background

After the introduction of quantum mechanics as a tool for describing nature the former concept of a continuous space-time had to be reconsidered. In a mathematical description by means of vector fields of particles and their interrelations we still make use of a continuous space-time, but when experiments are made in which location and time are measured, nature exhibits its discrete aspect.

For human beings it is obvious that between their ideas and materia there is a basic difference. Is this actually true? The complexity of our brain admits the thought that each idea of an intelligent being is related to precisely on material configuration in his brain.

Such ideas lead to a critical reconsideration of the concept of continuum. In particular we will investigate in what sense uncountable sets exist, for example the uncountable set of reals, considering the fact that in a discrete world everything that exists including the ideas of human beings has to be countable.

2. Communications on screems

The question arises how this "world" of ideas may be described such that the problems of countability and uncountability may be discussed. We will try to give such a description.

In order to make a first step in the direction of our problem a further restriction is necessary: We restrict the "world" to those ideas which can be "communicated" in the usual way. What does this mean? Ever since human beings worked scientifically, they wrote down the results of their researches and discussed them. Thus we will restrict our considerations to those ideas which can be written down.

Clearly, this excludes (a prima vista) the "world of emotions".

When we try to make statements on "all possible communications" we encounter the problem that it seems to be impossible to ever understand all ideas of all thinking beings. It thus seems to be necessary to abandon the concept of idea and to replace it by the concept of communications which, in principle, may be written down.

This restriction to written communications (the form of which will be discussed below) permits us to dispense the individuality of ideas. *Such communications are not ideas of our internal vision but objects that may be described by means of methods of science.*

We first will order all possible written communications. Our starting point is that it is always possible to represent any written communication by means of a TV-screen. In doing so we proceed as follows:

- First, we consider only square screens.
- Second, as basic unit of length for a screen we choose 1/10mm.
- Third, we consider a sequence of screens of sidelengths $1/10mm$, $2/10mm$, ...

Thus a screen of sidelength $n/10mm$ consists of n^2 basic squares.

- Fourth, for any screen of sidelength $n/10mm$ there are 2^{n^2} different black and white colourings of the n^2 basic squares of the screen.
- Fifth, each colouring of a screen is called a *"communication"* on the screen.
- Sixth, there are countably such communications. Consider a fixed linear ordering of these.
- Seventh, this ordering is called a *"universal ordering"*.

It may not seem appropriate to call any such colouring a *communication*. Whether a colouring is a communication in the usual sense can be recognized only by intelligent beings seeing it. The basic idea of our universal ordering is that its meaning is indpendent of those intelligent beings who see it.

3. An ordering of all statements that can be made by intelligent beings

In this section we relate the universal ordering and the statement that can be made by intelligent beings and which are representable on a screen. Since a countable linear ordering is required, we try to define such an ordering:

Subdivide our space-time world into fourdimensional space-time elements. Consisting of a cube with edgelength equal to the elementary unit length and lasting for the elementary unit time. Consider a (countable linear) ordering of (all finite sets of) space-time elements.

Next, each intelligent being has a certain space time extension. Whenever such a being makes a statement, the location and the time of it determine a finite set of space-time elements.

By combining communications (see section 2) and the corresponding space-time elements just considered, all possible statements of intelligent beings which are representable on a screen together with their communications on screens may be arranged in a countable linear ordering.

4. A linear ordering of all real numbers

We now select from this ordering those statements which determine real numbers. These are the statements, which induce in the intelligent being making the statement the thought "This communication defines in my opinion a real number".

These results must not be descriptions of reals for the reader but they describe the combinations of statements and communications on screens as related to intelligent being for which such a statement defines a real and for which the communication is a representation of the statement on a screen. *The meaning of the communication for the reader is of no importance.*

We observe that this ordering is not affected by the possibility that the statement may induce erroneously the thought that it defines a real. The ordering contains all correct statements as well as all erroneous ones and even those which are deliberately false. For our purpose it is decisive that it contains all possible correct definitions of reals. In other words, for each real which at a certain moment is defined by a communication of an intelligent being there is a well-defined position for it in this ordering.

In general a proof of the incompleteness of a countable linear ordering is by the specification of an element not contained in it. Thus there is a moment when an intelligent being — the critic of the completeness of the ordering — describes an element which is not contained in the ordering.

So far we have avoided to order elements. We have ordered statements (and their representations on screens) which in suitable intelligent beings induce the thought that the statement defines a real. Whether this is true is of no concern to us. We have not ordered reals, but statements of intelligent beings in which the latter make statements about reals.

We now assert that in this (countable linear) ordering there is a position for each real which does not correspond to any other real. A critic thus would have to make a statement that there is a real (which he can determine) for which there is no position in the ordering.

For this statement of an arbitrary critic at any moment we have in our ordering a position. (Since screens looked at from twice the original distance by doubling the edge length of the basic squares have the same image, there are even infinitely many communications for each statement, that is, infinitely many positions.) Since the statement of the critic can be represented in form of a communication for which (together with the moment and the location of the critic) there is reserved a position, this leads to a contradiction: The critic asserts at the same time that his statement defines a real for which there is no position in the ordering whereas his statement precisely by its meaning has a well-defined position in the ordering.

In the same way all those incompleteness proofs of a countable linear ordering of elements of a (fictively) uncountable set may be refuted in which the critic determines an element which is not contained in the ordering. Such a definition represented as a communication on a screen and combined with the location and moment of the critic leads to a position which is reserved for this element.

5. The diagonal method of Cantor

For the proof of the uncountability of the reals in the interval [0, 1] Cantor's diagonal method may be used as follows: In a given — hypothetical — countable linear ordering of the reals in [0, 1] write these in their decimal representation. Then define a further real as follows: If the nth digit of the nth real is 0, define the nth digit of this number to be 1, and 0 otherwise. This new real number is distict from any real number in the countable linear ordering since for any n the nth digit of the nth number in the linear ordering differs from the nth digit of the decimal expansion of the new number.

This method is not applicable to the ordering of free positions for "any real number". At present, resp. in the moment when a criticism is made it is not clear whether all decimal numbers for which there is a position in the ordering actually are definable. This is due to the fact that the ideas of all possible intelligent beings whether a certain statement defines a real number or not have to be known now. Aside from these arguments, the criticism per se leads to a contradiction. Any criticism can be represented in the form of a communication on a screen. This communication together with the location and the moment of the criticism has its position in the ordering. It is precisely the assertion of the critic, the "diagonal" number thus defined represents a real in the interval [0, 1] which makes sure that it has a position in the ordering. According to the definition of Cantor's diagonal number its digit which corresponds to its position in the ordering has to be different from precisely this digit. Thus the definition of the decimal number which was constructed to prove the incompleteness of the ordering is contradictory.

6. Contradictions in other proofs of uncountability

In analogy to the above derivation of a contradiction in the construction of a diagonal number which is to prove the uncountability of the set of reals, we may find a contradiction in the proofs of the impossibility of a countable linear ordering of all those — hypothetically — uncountable sets in which the contradiction is found by constructing an element which — hypothetically — is not contained in a linear countable ordering in the set.

Again the starting point is a countable linear ordering of statements of intelligent beings in which these beings assert in well defined moments that by the corresponding communications on screens an element of the set considered is described.

If a critic asserts that for a certain element of the set there is no position reserved, then he makes at a certain moment at a certain location a statement — which may be represented in the form of a communication on a screen — and which defines an element of the given set. But for this assertion of the critic there is reserved in the linear ordering a position. Thus the critic makes the following statement: For the element which I describe there is no position in the ordering, but it is precisely this statement of the critic which makes sure that there is a position for this element. Contradiction.

7. Once again: the universal ordering

By a combination of the countably many communications on screens with the countably many space-time elements we have found a countable linear ordering of all possible statements of any intelligent being at any moment. Clearly, there is a set equivalent to this set and contained in the universal ordering:

To show this we select those communications on screens which contain the following three parts:

1. A written communication of an arbitrary type.
2. The specification of a space-time element which determines an intelligent being.
3. The statement of the intelligent being that by the above communication for him

a unique element of the set considered is determined.

By the communications thus defined we obtain for each element of a set which can be thought of a position. The corresponding ordering defines a countable ordering of all elements of arbitrary sets. Any argument saying that this ordering is not complete and which is based on the assertion that there is no position reserved for a certain additional element, leads to a contradiction: as shown before, precisely this assertion of a critic guarantees that for this element there is reserved a position in the ordering.

Concluding, we remark that the linear countable orderings that we have proposed dont contain real numbers or other elements of given sets. The reals, resp. elements which corresponds to the positions are determined by the intelligent beings corresponding to a special space time element. The truth of the assertion of this intelligent being that the communication on the screen actually determines the element which the intelligent being has in mind is irrelevant for the ordering.

8. Formalization

Let $\{BSM\}$ be the set of all communications on screens. $\{BSM\}$ is countable and may be ordered linearly. Call a fixed such ordering a *universal ordering* (UO).
Let $\{RZE\}$ be the set of all space time elements.
Let $\{A\}$ be the set of all propositions which may be formulated by means of a BSM.
Define $A(E \in M)$ to be the proposition $E \in M$.

Denote by $B[A, RZE]$ a communication which says that a person which is uniquely described by RZE asserts A at a moment that is uniquely determined by RZE.

Arbitrary alements E of arbitrary sets M for which there is a proposition $A = A(E \in M)$ can be ordered in the following way:
The position of each BSM of the form $B[A(E \in M), RZE]$ in UO is related to E. (Obviously there are infinitely many positions for each E.)

$$B[A(E \in M), RZE] \to E.$$

Assertion:
To any element E of any set M there corresponds at least one position in UO which corresponds to no other element.
Corollary:
All sets of elements for which propositions $A(E \in M)$ may be formulated as BSM are countable.
Proof (by contradiction):
Assume, there is an $E \in M$ to which corresponds no position in UO.

$$\forall_{RZE} \exists \neg B[A(E \in M), RZE] \to E \qquad (*)$$

Then there is an intelligent being which can assert the latter in a certain moment. This intelligent being thus affirms the proposition $A(E \in M)$ in a certain moment.

Thus there is at least one RZE corresponding ot this person and this moment such that $B[A(\bar{E} \in \bar{M}), \bar{R}Z\bar{E}] \in UO$ and

$$B[A(E \in M), RZE] \to E$$

in contradiction to $(*)$.

9. Epilogue

We point out connections to Wittgenstein.

He indicated the final limitations of a meaningful language. In doing so he described the mystical resp. the transcedental from "the interior side".

Once Wittgenstein was asked where in his system a disdainful movement of the hand has its place. Since he could not give a definite answer this led him to further investigations.

Obviously, Wittgenstein considered the language as a means of transportation of information between persons. Thus besides the logical correctness of the sentences also the "rules" for the use (meaning?) of concepts is of importance. In our system UO of universal ordering the meaning of the information for a second person is irrelevant. Thus UO contains all sentences which are meaningful in Wittgenstein's sense but also other sentences. In the UO system one can speak about things about which in Wittgenstein's sense one cannot speak. Examples are expressions of emotions such as a disdainful movement of the hand. These are describe by the space time element where they take place.

Possibly one can say, the UO system contains in addition to all sentences which are meaningful in Wittgenstein's sense also the sentences which are meaningful for the person in the moment when he or she says them. This covers the transcendental and the mystical in Wittgenstein's sense.