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SEMINAR PAPER

Loss Coverage : Why Insurance Works Better With Some Adverse Selection

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Juli 30, 2019

Abstract

The following seminar paper is based on the book “Loss Coverage : Why Insurance Works Better With Some Adverse Selection”, written by Guy Thomas and published in 2017. For some reason the book caught my attention by the title, so I began my investigation of this book, reading it and pondering over its contents. It offered me something more than I had previously known about adverse selection. The main focus of the book is the argument that some restrictions on risk classification, far from having adverse effects, can actually make insurance work better, in the sense of increase loss coverage. Loss coverage is defined as the expected losses compensated by insurance. Personally, I find it’s a very well written and interesting book.

Contents

1 Introduction

1.1 What is adverse selection?

Adverse selection is a term commonly used in economics, insurance, and risk management that describes a situation where market participation is affected by asymmetric information. Information asymmetry is known as a state that "When buyers and sellers have different information"

Why is it something to be avoided at all cost based on the orthodox views?
In the orthodox views that held by the insurance industry, actuaries and economists, this asymmetry creates an imbalance of power in transactions, which can sometimes cause the transactions to go awry, a kind of market failure in the worst case.

A succinct statement of this orthodoxy is given in the policy document "Insurance @ superannuation risk classification policy" published by the institute of Actuaries in Australia, which explains:

"In the absence of a system that allows for distinguishing by price between individuals with different risk profiles, insurers would provide an insurance or annuity product at a subsidy to some while overcharging others. In an open market, basic economics dictates that individuals with low risk relative to price would conclude that the product is overpriced and thus reduce or possibly forgo their insurance. Those individuals with a high level of risk relative to price would view the price as attractive and therefore retain or increase their insurance. As a result the average cost of the insurance would increase, thus pushing prices up. Then, individuals with lower loss potential would continue to leave the marketplace, contributing to further price spiral. Eventually the majority of consumers, or the majority of providers of insurance, would withdraw from the marketplace and the remaining products would become financially unsound." [1]

Adverse selection spiral
Adverse selection spiral or "death spiral" is known as the sequence of a rise in insurance prices and fall in numbers insured, followed by a further rise in insurance prices and fall in numbers insured, and so on .

1.2 Scope & Focus

The focus is on Personal Insurances, particularly those contingent in some way on the insured's life and health (life insurance, annuities, income protection insurance, critical illness insurance and health insurance (medical expenses insurance)). For these insurance, higher risk often face not only prospective disadvantage, but also some degree of current disadvantage (e.g some degree of current ill-health). To a lesser extent, he also have in mind other personal insurances, such as travel, home and car insurance.

[1] Loss Coverage: Why Insurance Works Better With Some Adverse Selection, p.3-4

The insurances where the insured is a corporation of comparable strategic sophistication to the insurer, or where the insured views the contract as part of a speculative investment portfolio, rather than as protection against some unlikely and undesirable contingency is not in the consideration and the intuitions about public policy, and particularly perceptions of fairness in risk classification, are highly sensitive to this scope and focus.

2 Introduction to The Argument

2.1 Adverse Selection and Loss Coverage

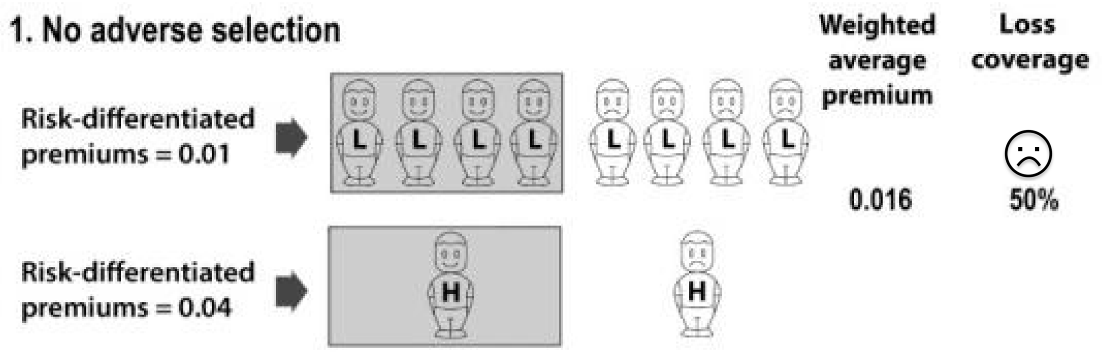
The main innovation is the concept of **"Loss Coverage".** The impact of adverse selection should be measured, not in terms of the number of insured, but rather in terms of loss coverage.
Consider an insurance market where individuals can divided into two risk-groups, one higher risk and one lower risk, based on the information which is fully observable by insurers. Assume that all losses and insurance are of unit amount. Also assume that an individual's risk is unaffected by the purchase of insurance, i.e there is no moral hazard. If insurers can, they will charge risk-differentiated prices to reflect the different risks. If instead the insurers are banned from differentiating between higher and lower risks, and have to charge a single pooled price for all risks, a pooled price equal to the simple average of the risk-differentiated prices will seem cheap to higher risks and expensive to lower risks. Higher risks will buy more insurances, and lower risks will buy less. To break even, insurers will then need to raise the pooled price above the simple average of the prices. Also, since the number of higher risks is typically smaller than the number of lower (or standard) risks, higher risks buying more and lower risks buying less implies that the total number of people insured usually falls.This combination of a rise in price and a fall in demand is usually portrayed as a bad outcome, for both insurers and society.
However, from the social perspective, it is arguable that higher risks are those more in need of insurance. Also, the compensation of many types of loss by insurance appears to be widely regarded as a desirable objective, which public policymakers often seek to promote by public education, by exhortation and sometimes by incentives such as tax relief on premiums. Insurance of one higher risk contributes more in expectation to this objective than insurance of one lower risk.
This suggests that public policymakers might welcome increased purchasing by higher risks, except for the usual story about adverse selection.
The usual story about adverse selection overlooks one point: with a pooled premium and adverse selection, loss coverage can still be higher than with fully risk-differentiated premiums and no adverse selection. Although pooling (pooled price) leads to a fall in numbers insured, it also leads to a shift in coverage towards higher risks. From the public policymaker's viewpoint, this means that more of the 'right' risks - those more likely to suffer loss - buy insurance. If the shift in coverage is large enough, it can more than outweigh the fall in numbers insured.
This result of higher loss coverage can be seen as a better outcome for society than that obtained with no adverse selection.

2.2 Toy Example

The argument above can be illustrated by this toy example. Consider a population of just ten risks (lives), with three scenarios for adverse selection.
The first scenario, risk-differentiates prices are charged, and a subset of the population buys insurance.(no adverse selection). The second and third scenarios, risk classification is banned, leading to adverse selection: a different subset of the population buys insurance.
In Figure 1-3, each ‘H’ represents one high risk and each ‘L’ represents one low risk. The Population has typical predominance of lower risks : eight lower risks each with probability of loss 0.01, and two higher risks each with probability of loss 0.04.
In each scenarios, the shaded ‘cover’ above some ‘H’ and ‘L’ denotes the risks covered by insurance.

Cover = The risks covered by insurance.

Figure 1 : High and low risks covered in same proportions as in population
=> No adverse selection (base outcome)



Scenario 1
In Scenario 1 (no adverse selection), in Figure 1, risk-differentiated premium are charged. Higher and lower risk-group each face a price equivalent to their prob-ability of loss (an actually fair price). The demand response of each risk-group to an actually fair price is the same: exactly half the members of each group buy insurance. The shading shows that a total of five risks are covered. Note that the equal areas of shading over one ‘H’ and four ‘L’ represent equal expected losses.

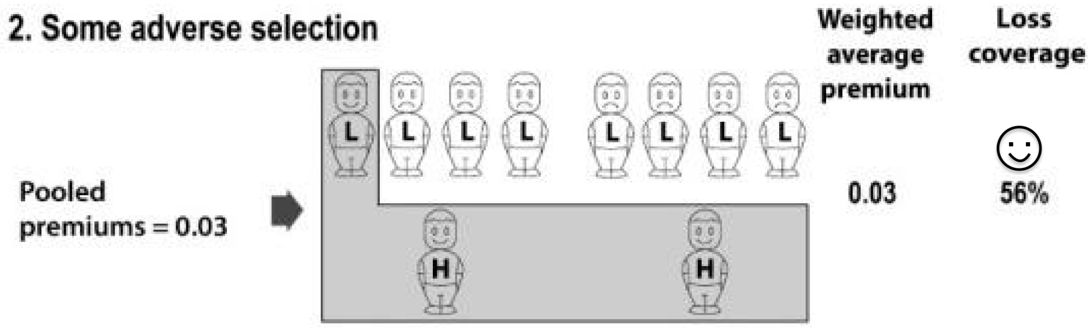
The Weighted Average of the Premiums : $\dfrac{(4 \times 0.01) + (1 \times 0.04)}{5} = 0.016.$

Since higher and lower risks are insured the same proportion as they exist in the population, there is no adverse selection.

The expected losses compensated by insurance for the whole population can be indexed by:

$$\textbf{Loss Coverage} = \frac{(4 \times 0.01) + (1 \times 0.04)}{(8 \times 0.01) + (2 \times 0.04)} = 50\,\%$$

Figure 2 : Higher weighted average premium, lower numbers insured
=> Moderate adverse selection



The shift in coverage towards higher risks more than offsets lower numbers insured => higher loss coverage (better outcome)

Scenario 2
In Scenario 2 (moderate adverse selection), in Figure 2, risk classification has been banned, and so insurers have to charge a common 'pooled' premium to both higher risks and lower risks. Higher risks buy more insurance and lower risks buy less. The shading shows that three risks (compared with five previously) are now covered. The pooled premium is set as the weighed average of the true risks, so that expected profits on low risks exactly offset expected losses on high risks.

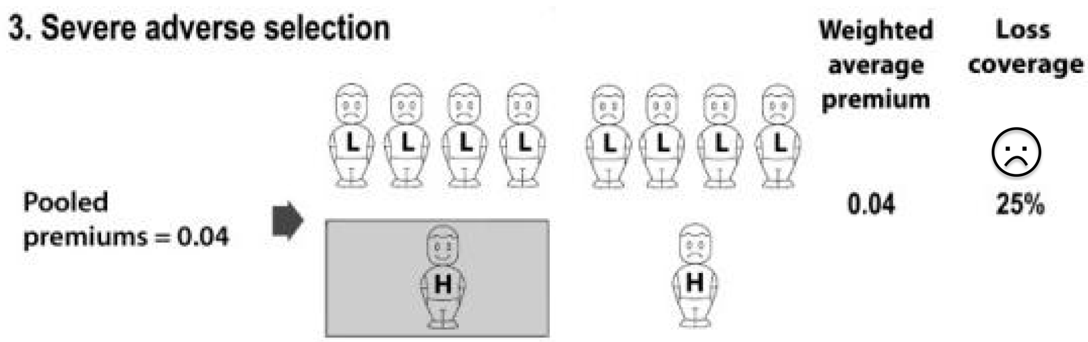
$$\textbf{This Weighted Average of the Premiums is :} \quad \frac{(1 \times 0.01) + (2 \times 0.04)}{3} = 0.03$$

Note that the weighted average premium is higher in Scenario 2, and the number of risks insured is smaller. These are the essential features of adverse selection, which Scenario 2 accurately and completely represents. But there is a surprise: despite the adverse selection in Scenario 2, the expected losses compensated by insurance for the whole population are now larger. Visually, this is represented by the larger are of shading in Scenario 2 .

The loss coverage in Scenario 2 is:

$$\textbf{Loss Coverage} = \frac{(1 \times 0.01) + (2 \times 0.04)}{(8 \times 0.01) + (2 \times 0.04)} = 56\,\%$$

Figure 3 : Only one individual (higher risk) remains insured
=> Severe adverse selection



Shift in coverage towards higher risks does not offset lower numbers insured
=> lower loss coverage (worse outcome)

Scenario 3
A ban on risk classification can also reduce loss coverage, if the adverse selection which the ban induces becomes too severe. This possibility is illustrated in Scenario 3 (Severe adverse selection). Adverse selection has progressed to the point where only one higher risk, and no lower risks, buys insurance. The expected losses compensated by insurance for the whole population are now lower. That is, 25% of the population's expected losses are now compensated by insurance, compared with 50% in Scenario 1, and 56% in Scenario 2.

Summary
The key idea is that loss coverage is increased only by the "right amount" of adverse selection, but reduced by "too much" adverse selection. Which of Scenario 2 or Scenario 3 actually prevails depends on the demand elasticities of higher and lower risks.

3 The Argument

In this chapter, more detailed and realistic numerical examples than the toy examples will be given, showing that while loss coverage is increased by 'the right amount' of adverse selection, it can be reduced if there is 'too much' adverse selection. " The Argument that some degree of adverse selection makes the insurance system work better " will be presented in three way ; 3.1 the key argument, 3.2 numerical examples and 3.3 mathematical details.

3.1 The Key Argument

The key argument repeats the first part of "Adverse Selection and Loss Cover-age". Another perspective on this argument is that a public policymaker designing risk classification policies in the context of adverse selection normally faces a trade-off between insurance of the 'right' risks and insurance of a large number of risks. The optimal trade-off depends on the response of higher and lower risk-groups to different prices, (technically, the demand elasticity of different risk-groups), indwell normally involve a least some adverse selection. The concept of loss coverage quantifies this trade-off, and provides a metric for comparing the effects of different risk classification schemes.

3.2 Numerical Examples

Numerical examples are similar in nature to the three scenarios illustrating 'no adverse selection', 'some adverse selection' (moderate adverse selection) and 'too much' adverse selection (severe adverse selection) in the toy example above. Suppose that,

- a population of 1000 risks,
- 16 losses are expected every year
- there are 2 risk-groups
- 200 high risks
- 800 low risks
- the high risks have a probability of loss 4 times higher than the low risks.
- We assume that all losses and insurance are of unit amount, and that there is no moral hazard. An individual's risk-group is fully observable to insurers.

Table 1 Full Risk-Classification: no adverse selection (base outcome)

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1 000
Expected population losses	8	8	16
Break-even premiums (risk-differentiated)	0.01	0.04	0.016
Numbers insured	400	100	500
Insured losses	4	4	8
Adverse selection			1
Loss coverage			0.5

Table 2 Risk-Classification banned : moderate adverse selection leading to higher loss coverage (better outcome)

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1 000
Expected population losses	8	8	16
Break-even premiums (pooled)	0.02	0.02	0.02
Numbers insured	300	150	450
Insured losses	3	6	9
Adverse selection			1.25
Loss coverage			0.5625

Table 3 Risk-Classification banned: severe adverse selection leading to lower loss coverage (Worse outcome)

	Low risk-group	High risk-group	Aggregate
Risk	0.01	0.04	0.016
Total population	800	200	1 000
Expected population losses	8	8	16
Break-even premiums (pooled)	0.02154	0.02154	0.02154
Numbers insured	200	125	325
Insured losses	2	5	7
Adverse selection			1.34625
Loss coverage			0.4375

Under the initial risk classification regime, insurers operate full risk classification charging actuarially fair premiums to members of each risk-group. We assume that the proportion of each risk-group which buys insurance under these conditions - the ‘ fair-premium demand’ - is 50% which is realistic for life insurance in the UK and the USA (base on the Author research).

The Table 1 shows the outcome, which can be summarised as follows: There is no adverse selection. The average of the insurance premiums, weighted by numbers of insurance buyers at each price, is 0.016 (final column, fourth line). This is the same as the population-weighted average risk (final column, first line). Dividing the first by

the second, we index the adverse selection as $\frac{0.016}{0.016} = 1$, indicating a neutral position (i.e. no adverse selection). Half the losses in the population are compensated by insurance. We heuristically characterise this as a 'loss coverage' of 0.5.
Now suppose the a new risk classification regime is introduced, where insurers are obliged to charge "a common 'pooled' premium" to members of both the low and high risk-groups.
One possible outcome is show in Table 2, which can be summarised as follows: The pooled premium of 0.02 at which insurers make zero profits is calculated as the demand-weighted average of risk premiums: $\frac{(300 \times 0.01) + (150 \times 0.04)}{450} = 0.02$. The pooled premium is expensive for low risks, so 25% fewer of them buy insurance (300, compared with 400 before). The pooled premium is cheap for high risks, so 50% more of them buy insurance (150, compared with 100 before). Because there are 4 times as many low risks as high risks in the population, the total number of policies sold falls (450, compared with 500 before). There is moderate adverse selection. The pooled premium of 0.02 exceeds the population-weighted average premium of 0.016, giving adverse selection: $\frac{0.02}{0.016} = 1.25$. The resulting loss coverage is 0.5625. The shift in coverage towards high risks more than outweighs the fall in number of the policies sold: 9 of 16 losses (56%) in the population as a whole are now compensated by insurance (compares with 8 of 16 before).Another possible outcome under the restricted risk classification scheme the time with more severe adverse selection, is shown in Table 3 which can be summarised as follows: The pooled premium of 0.02154 at which insurers make zero profits is calculated as the demand-weighted average of the risk premiums: $\frac{(200 \times 0.01) + (125 \times 0.04)}{325} = 0.02154$. There is severe adverse selection, with further increase in the pooled premium and a significant fall in numbers insured. The loss coverage is 0.4375. The shift in coverage towards high risks is not sufficient to outweigh the fall in number of policies sold : 7 of 16 losses (43.75%) in the population as a whole are now compensated by insurance (compared with 8 of 16 in Table 1, and 9 out of 16 in Table 2).
Taking the three tables together, we cam summarise by saying that compared with an initial position of no adverse selection in Table 1, moderate adverse selection leads to higher expected losses compensated by insurance (higher loss coverage) in Table 2 , but too much adverse selection leads to lower expected losses compensated by insurance (lower loss coverage) in Table 3.
This argument that moderate adverse selection increases loss coverage is quite general: it does not depend on any unusual choice of numbers for the examples.
It also does not assume any bias by the policymakers towards (or against) compensating the losses of the high risk-group in preference to those of low risk-group. The same preference is giving to compensation of losses anywhere in the population ex post, when all uncertainty about who will suffer a loss has been resolved. This implies giving higher preference to insurance cover for higher risks ex ante, before we now who will suffer a loss, but only in proportion to their higher risk.

Summary

This way it has suggested that the usual arguments that adverse selection alway makes insurance work less well, and that more adverse selection is always worse than less, are misconceived.Adverse selection implies a fall in numbers insured, and also a shift in coverage towards higher risks. If the shift in coverage outweighs the fall in numbers, the expected losses compensated by insurance are increased. From a public policy perspective, a degree a so-called ‘adverse’ selection in insurance is a good thing.

3.3 Mathematical Details

The discussion before was informal. Now let’s look at Mathematical Details , since we are mathematics students. In mathematical details will be divided it in two parts, which gonna be ,3.3.1 Basic Mathematics of Loss Coverage and 3.3.2 Further Mathematics of Loss Coverage.
Throughout these, we assume a population of risks can be divided into low risk-group and high risk-group, based of information which is fully observable to insurers.
Just two risk-groups is of course not a realistic model of most insurance markets, but it is enough to illustrate principles. [2]
We also assume that all losses and insurance cover are of unit size, and no moral hazard, that is, giving the risk-group, the probability of loss is not affected by the purchase of insurance.

3.3.1 Basic Mathematics of Loss Coverage

A Model for an Insurance Market
Let μ_1 and μ_2 be the probabilities of loss for the low and high risk-groups.
Let p_1 and p_2 the the population fractions for the low and high risk-groups, that is the proportions of the total population represented by each risk-group.
This means a risk chosen at random from the entire population has a probability p_1 of belonging to the low-group.

All quantities defined below are for a single risk sampled at random from the population (unless the context requires otherwise).

The Expected Loss is denoted by $\mathbb{E}[L]$ (L for Loss) and given by :

$$\mathbb{E}[L] = \sum_{i=1}^{2} \mu_i p_i \qquad 1.1$$

$\mathbb{E}[L]$ corresponds to a unit version of the third row of the table 1-3.

In absence of limits on risk classification, insurers will charge risk-differentiated premiums equal to the probabilities of loss, $\pi_1 = \mu_1$ and $\pi_2 = \mu_2$ for risk-groups 1 and 2, respectively.

The Expected Insurance Demand is denoted by $\mathbb{E}[Q]$ (Q for Quantity) and given by:

$$\mathbb{E}[Q] = \sum_{i=1}^{2} d(\mu_i, \pi_i) p_i \qquad 1.2$$

[2] (The extension to N risk-groups is straightforward, see for example Hao et al. [2016b])

where $d(\mu_i, \pi_i)$ is the proportional demand for insurance for risk-group i when premium π_i is charged, that is the probability that an individual selected at random from the risk-group buys insurance.

The Expected Premium is denoted by $\mathbb{E}[\Pi]$ (Π for Premium) and given by :

$$\mathbb{E}[\Pi] = \sum_{i=1}^{2} d(\mu_i, \pi_i) p_i \pi_i \tag{1.3}$$

The Expected Insurance Claim is denoted by $\mathbb{E}[QL]$ and given by :

$$\mathbb{E}[QL] = \sum_{i=1}^{2} d(\mu_i, \pi_i) p_i \mu_i \tag{1.4}$$

Adverse Selection

The phrase 'adverse selection' is typically associated with positive correlation (or equivalently, covariance) of cover Q and loss L; most papers testing for adverse selection in economics literature use this definition. "This section makes this definition precise, in a form which will later help to highlight the relationship between advise selection and loss coverage."

By the standard definition, the Covariance of Q and L is :

$$Cov(Q, L) = \mathbb{E}[(Q - \mathbb{E}[Q])(L - \mathbb{E}[L])] = \mathbb{E}[QL] - \mathbb{E}[Q]\mathbb{E}[L] \tag{1.5}$$

While the economics literature typically uses covariance $(Q, L) > 0$ as a test for adverse selection, it is more convenient to note that when the covariance is zero, the two term in Equation (5) must be the same.

Then Adverse Selection is denoted by A and defined as :

$$A = \frac{\mathbb{E}[QL]}{\mathbb{E}[Q]\mathbb{E}[L]} \tag{1.6}$$

This enable us to index different types of selective behaviour by insurance customers as follows :

$A < 1$: advantageous selection
A = 1 : no selection
A > 1 : adverse selection

(See in Tables 1-3)

To compare the severity of adverse selection under different risk-classification regimes, we need to define a reference level of adverse selection.
Adverse selection under alternative schemes can then be expressed as a fraction of adverse selection under the reference scheme.
A convenient reference scheme is risk-differentiated premiums (actuarially fair

premiums).
Then using subscript 0 to denote quantities evaluated under risk-differentiated premiums, we define the adverse selection ratio as :

$$\text{Adverse selection Ratio} = \frac{A}{A_0} \qquad 1.7$$

In words, adverse selection ratio is the ratio the expected claim per policy under the actual risk classification scheme to the expected claim per policy under risk-differentiated premiums.
Adverse Selection ratio can also be thought of as the ratio of the demand-weighted average premiums required for insurance to break even under each risk classification scheme.

Loss Coverage

Loss Coverage in this book is defined as the expected insurance claim (as previously evaluated in equation (1.4)) :

$$\text{Loss Coverage} = \mathbb{E}[QL] = \sum_{i=1}^{2} d(\mu_i, \pi_i) p_i \mu_i \qquad 1.8$$

The product of random variables Q and L can alternatively be thought of as the following ‘ indicator’ random variable:

$$QL = \{1 \text{ if the individual both incurs a loss and has cover, 0 otherwise}\} \qquad 1.9$$

Loss Coverage can then be thought of as indexing the ‘overlap’ of cover Q and losses L in the population. It represent the extent to which insurance cover is concentrated over the ‘ right’ risks (those most likely to suffer loss). It measures the efficacy of insurance in compensating the population’s losses.
The right-hand side of Equation (1.8) also shows that loss coverage can be contrasted with unweighted insurance demand, which corresponds to the ‘number of risks insured’ often referenced in information discussions of adverse selection.

Loss Coverage Ratio

When comparing alternative risk classification schemes, it is often helpful to define loss coverage to be 1 under some suitable reference scheme. Loss Coverage under alternative schemes can then be expressed as a fraction of loss coverage under the reference scheme.
It is convenient to use the same approach as for adverse selection above, that is, I use risk-differentiated premiums as the reference scheme.
Then using subscript 0 the denote quantities evaluated under risk-differentiated premiums, I define the loss coverage ratio (LCR) as :

$$LCR = \frac{\mathbb{E}[QL]}{\mathbb{E}_0[QL]} \qquad 1.10$$

Note that in the numerical examples in Chapter 1 and 3 , the fraction between 0 and 1 which I heuristically labelled 'loss coverage' was, more precisely, a loss coverage ratio with the reference scheme (i.e under which loss coverage = 1) defined as compulsory insurance of the whole population. Clearly the choice of reference scheme - risk-differentiated premiums, compulsory insurance or something else - does not matter, provided we use a consistent reference when making comparisons of different proposed risk classification schemes.
Now note that loss coverage ratio in Equation (1.10) can also be expanded into:

$$LCR = \frac{\left(\frac{\mathbb{E}[QL]}{\mathbb{E}[Q]\mathbb{E}[L]}\right)}{\left(\frac{\mathbb{E}_0[QL]}{\mathbb{E}_0[Q]\mathbb{E}[L]}\right)} \times \frac{\mathbb{E}[Q]}{\mathbb{E}_0[Q]} \qquad 1.11$$

Then by noting that expected population losses are the same irrespective of risk classification scheme (i.e. $\mathbb{E}[L] = \mathbb{E}_0[L]$), we see that the first term on the right-hand side of Equation (1.11) i the adverse selection ratio in Equation (1.7). The second term on the right-hand side of Equation (1.11) is the ratio of demand under the actual risk classification scheme to demand under risk-differentiated premiums, which I call the demand ratio.
So Equation (1.11) can then be interpreted as:

$$LCR = \text{Adverse Selection Ratio} \times \text{Demand Ratio} \qquad 1.12$$

Table 4 : Decomposition of loss coverage ratio into adverse selection rationed demand ratio for Tables 1 - 3.

	Table 1	Table 2	Table 3
Adverse selection ratio: $\left(\frac{\text{risk-weighted average premium}}{\text{population-weighted average premium}}\right)$	1.0	1.25	1.3461
Demand ratio: $\left(\frac{\text{numbers insured under pooled premium}}{\text{numbers insured under risk-differentiated premiums}}\right)$	1.0	0.90	0.65
Loss coverage ratio (product of above): $\left(\frac{\text{loss coverage under pooled premium}}{\text{loss coverage under risk-differentiated premiums}}\right)$	1.0	1.125	0.875

We can illustrate this decomposition of loss coverage ratio by applying it to the numerical examples from above. The decomposition is shown in Table 4. In each of the three columns in the table, loss coverage ratio (the third line) is the product of

adverse selection ratio and demand ratio (the first and the second lines).
The decomposition of loss coverage ratio into adverse selection ratio and demand ratio is sometimes helpful in correcting casual intuitions and commentary about restrictions on risk classification. Casual intuitions and commentary often reference rising average prices (adverse selection) and falling demand (numbers insured), but without considering how the two effects interact. But depending on the product of two effects, loss coverage ratio may be higher or lower than 1 (that is, loss coverage may be increased or decreased). The decomposition highlight that predicting or observing a rise in average price and fall in demand und a new risk classification scheme is not sufficient to demonstrate a worse outcome. The outcome in term of loss coverage depends on the product of two effects.

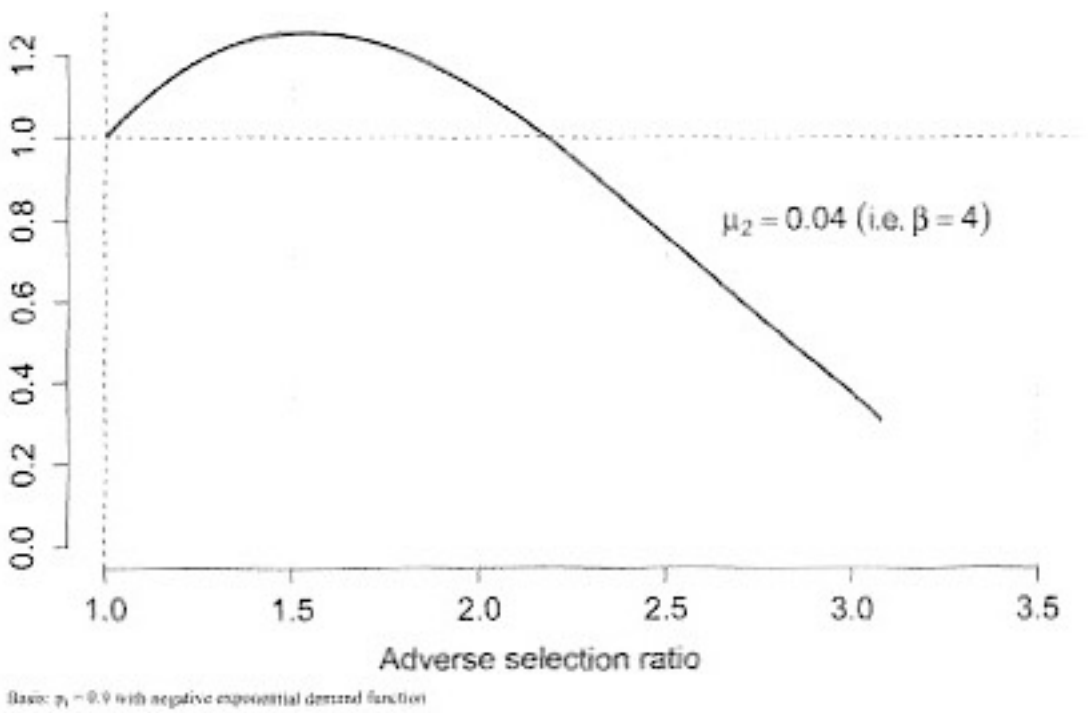


Figure 1.1: Loss Coverage ratio as a function of adverse selection ration

An illustration of the trade-off between loss coverage and adverse selection is provided by Figure 1. This graph plot loss coverage ratio against adverse selection ratio, based on the two risk-group model in this chapter with plausible form for the demand function. It can be seen that the maximum point for loss coverage corresponds to an intermediate degree of adverse selection, not too low and not too high. The shape of this graph, with the interior maximum showing that loss coverage is maximised by an intermediate level of adverse selection, is the most important image in this book. A similar invited U shape is obtained for any reasonable demand function.Note that highest loss coverage is obtained not despite the adverse selection, but because of the adverse selection. In moderation, adverse selection is a good thing. figure 1.1 is based on a relative risk $\beta = \frac{\mu_2}{\mu_1} = 4$. If the relative risk is lower, the maximum value of loss coverage is lower, and this maximum is attained with the lower level of adverse selection . This is illustrated in Figure 2 , which shows the plot of loss coverage ratio against adverse selection ratio for two values of relative risk, $\beta = 3$ and $\beta = 4$. The maximum of the dashed curve for $\beta = 3$ lies below and to the left on the maximum of the solid curve for $\beta = 4$.

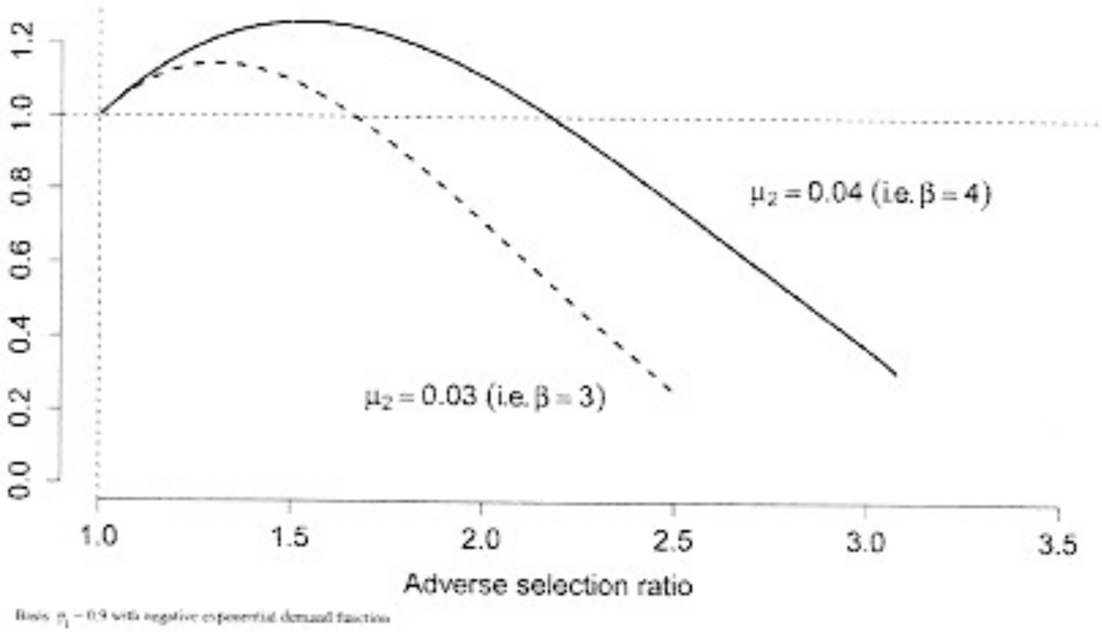


Figure 1.2: Loss Coverage ratio as a function of adverse selection ration, for two relative risks.

Note the the right-hand terminal points of each curve in figure 1.1 and 1.2 corres-pond to limiting values, not point at which I arbitrarily chose to stop drawing the curve.These limiting values represent the scenario where all lower risks have dropped out of insurance and only higher risk remain; clearly, adverse selection that cannot increase any more. In figure 1.2, the terminal point for $\beta = 3$ lies to the left and below the terminal point for $\beta = 4$. To understand this, note that when all the higher risks have dropped out the market and only lower risks remain, lower relative risk $\beta = 3$ implies a lower break-even premiums (lower adverse selection), and also that a lower fraction of the total risk in the population is covered (lower loss coverage).

3.3.2 Further Mathematics of Loss Coverage

The previous part of mathematical details gave mathematical definitions of loss cover-age and related quantities. This part introduces models of insurance markets which enable us to study how loss coverage varies with changes in the fraction of the popu-lation represented by higher risk and lower risks, probabilities of loss and demand elasticities. In this part we also use the same two risk-group model as in previous part.

We focus first on a simple iso-elastic demand function, and then consider more general demand functions.

Two Risk-Group with Iso- elastic Demand

In the previous part, a generic function for proportion of the population insurance demand $d(\mu_i, \pi_i)$ was used to define the quantities $\mathbb{E}[L]$ (expected population loss), $\mathbb{E}[Q]$ (expected insurance demand), $\mathbb{E}[\Pi]$ (expected premium), and $\mathbb{E}[QL]$ (expected claim (Loss Coverage)). A form for the demand function was not specified, nor detail of how the premiums π_i charged to each risk-group were determined. In this part, a form for the demand function will be specified. The zero-profit equilibrium condition which determines the 'pooled' premium when all risks are pooled at the same price also will be specified.

Specifying an Insurance Demand Function
To recap, the proportional demand for insurance $d(\mu_i, \pi_i)$ is the proportional of the risk-group the the probability of loss μ_i which buys insurance when a premium of π_i is charged to members of that risk-group.
As a preliminary, it is helpful to define the concept of demand elasticity.

Demand Elasticity is defined as :

$$\text{Demand elasticity} = \frac{\pi_i}{d(\mu_i, \pi_i)} \cdot \frac{\partial d(\mu_i, \pi_i)}{\partial \pi_i} \qquad 2.1$$

Roughly speaking, this is the percentage change in demand for a very small percentage change in premium. It measures the responsiveness of demand to small changes in premium.
Note that the derivative within the above expression in normally negative (as the premium rises, demand falls). The minus sign ensure that demand elasticity as defined here will normally be positive; this makes the subsequence mathematical presentation tidier.
It is sometimes more convenient to rewrite Equation (2.1) as the ***log-log derivative*** of the demand function with respect to the premium, that is :

$$\text{Demand elasticity} = -\frac{\partial \ \log[d(\mu_i, \pi_i)]}{\partial \ \log \pi_i} \qquad 2.2$$

What properties should the demand function possess?

(a) Decreasing in Premium: $d(\mu_i, \pi_i)$ is decreasing function of the premium π_i for all risk-groups.
(b) Increasing in Risk : $d(\mu_1, \pi_0) < d(\mu_2, \pi_0)$, that is at any given premium π_0 the proportional demand function is higher for the higher risk-group.
(c) Decreasing in Premium loading : $d(\mu_i, \pi_i)$ is decreasing function of the premium loading $\frac{\pi_i}{\mu_i}$.

(d) Capped at 1 : $d(\mu_i, \pi_i) \leq 1$, that is the highest possible demand is when all members of the risk-group buy insurance.

A simple demand function which satisfies these requirements can be obtained by the demand elasticity in Equation 2.2 equal to a constant, say λ_i. Solving this differential equation leads to the so-called i**so-elastic demand function** :

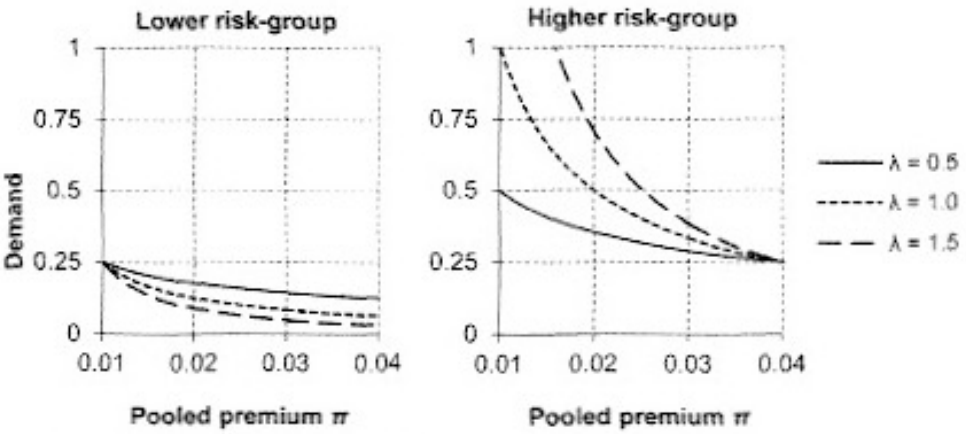
$$d(\mu_i, \pi_i) = \tau_i(\frac{\pi_i}{\mu_i})^{-\lambda_i} \qquad 2.3$$

- $\tau_i = d(\mu_i, \pi_i)$ is **the fair-premium demand** for the risk-group i , that is the proportion of the risk-group i who buy insurance at an actuarially fair premium, that is when $\pi_i = \mu_i$.
- μ_i is t**he risk (probability of loss)** for members of risk-group i.

λ_i is **the demand elasticity** for members of the risk-group i, as already defined.

To interpret the demand formula in Equation (2.3) , observe that it specifies demand as a function of the premium loading $(\frac{\pi_i}{\mu_i})$. When the premium loading is high (insurance is expensive), demand is low and vice versa. The 'iso-elastic' terminology reflects that price elasticity of demand is the same constant λ_i everywhere along the demand curve.

Figure 2.1: Iso-elasticity demand curves for $\lambda = 0.5,1,1.5$.



Basis: $p_1 = 0.9$; $\tau_1 = \tau_2 = 0.25$; $\mu_1 = 0.01$, $\mu_2 = 0.04$.

The λ_i parameter controls the shape of the demand curve. This is illustrated in Figure 2.1, which shows plots of demand from higher and lower risk-groups for three different values of λ in the demand function of Equation (2.3).

The demand function in Equation (2.3) clearly satisfies axioms (a) and (c) above. Axioms (b) and (d) appear superficially to require conditions on the fair-premium demands τ_1 and τ_2. In other words, we need to be careful that modelled demand from the lower risk-group is always lower than from the higher risk-group (axiom (b)) ; and also that modelled demand from the higher risk-group does not exceed 1 at the equilibrium premium (axiom (d)).

In Figure 2.1, we can see that the latter point might be a concern for the highest curve in the right panel, the case $\lambda = 1.5$, if the equilibrium-pooled premium happened to be below about 0.016.
However, for the purposes of analysing the mathematical properties of the model, it is convenient to use the following trick Recall that p_1 and p_2 are the fractions of the total population represented by lower risks and higher risk, respectively. Then define the **fair-premium demand-share** of the lower risk-group as the proportion of total demand which the risk-group represents when actuarially fair premiums are charged to both risk-groups.
Fair-Premium Demand-Share :

$$\alpha_i = \frac{\tau_i p_i}{\tau_1 p_1 + \tau_2 p_2}, \qquad i = 1,2 \qquad 2.4$$

Clearly α_2, the fair-premium demand-share of the higher risk-group, is just the complement of α_1 (i.e. $\alpha_2 = 1 - \alpha_1$).
We can then analyse the mathematical properties of the model for the full range of possible fair-premium demand-shares $0 \leq \alpha_1 \leq 1$, without worrying about specifying any particular values for the fair-premium demands τ_i and the population fractions p_i. It suffices to note that for every possible α_1, there will be **some** hypothetical combination of population structure p_i and fair-premium demand τ_i which satisfies the axioms (b) and (d) above.

Specifying a Zero-Profit Equilibrium Condition
Suppose now that the premiums charged to members of the low risk-group and high risk-group are π_1 and π_2, respectively.
The Insurance income (Premiums) per member of the population will be the sum of the products of demand and premium for each risk-group:

$$\text{Insurance Income} = d(\mu_1, \pi_1)p_1\pi_1 + d(\mu_2, \pi_2)p_2\pi_2 \qquad 2.5$$

The Insurance Outgo (Claims) per member of the population will be the sum of the products of demand and the probability of loss for each risk-group:

$$\text{Insurance Outgo} = d(\mu_1, \pi_1)p_1\mu_1 + d(\mu_2, \pi_2)p_2\mu_2 \qquad 2.6$$

The right-hand side Equation (2.6) looks like the definition of Loss Coverage in Equation (1.8) in the previous part. However, loss coverage refers specifically to this quantity at equilibrium. The insurance outgo in Equation (2.6) is defined for any premiums, not just equilibrium premiums.
To determine equilibrium premiums, note that the insurer's expected profit (loss, if negative) is expected income less expected outgo, that is Equations (2.5) - (2.6). So any pair of premiums (π_1, π_2) which equates Equation (2.5) and Equation (2.6) (=), is an Equilibrium.
One equilibrium is obvious from inspection: full risk classification (or 'full risk-differentiated premiums'), that is $\pi_1 = \mu_1$, $\pi_2 = \mu_2$. This represent one extreme.
Another equilibrium - and the main focus in this part - is a common 'pooled' premium π_0 for both risk-groups: nil risk classification (or 'pooling') that is $\pi_1 = \pi_2 = \pi_0$. This

represents the other extreme.
There will always exist some value of π_0 which gives a pooling equilibrium.

Examples
To illustrate the use of the model, set $\mu_1 = 0.01$, $\mu_2 = 0.04$ (i.e. relative risk $\beta = 4$) and $\alpha_1 = 0.9$ that is 90% of the insurance demand under the risk-differentiated premiums is from lower risks. These parameter values are used throughout this part of mathematical details, except where stated otherwise. The parameters have been chosen by loose analogy with the life insurance market, where typically around 90% of accepted risks are assigned to large 'standard' risk-group with low mortality, and around 10% of accepted risks are charged a range of higher premiums ranging from +50% to +300% over the standard risk-group's premium. But these values merely hypothetical and illustrative; they are not presented as a calibrated model of any real market.

Figure 2.2 : Low Elasticity: $\lambda = 0.5$, giving increased loss coverage under pooling.

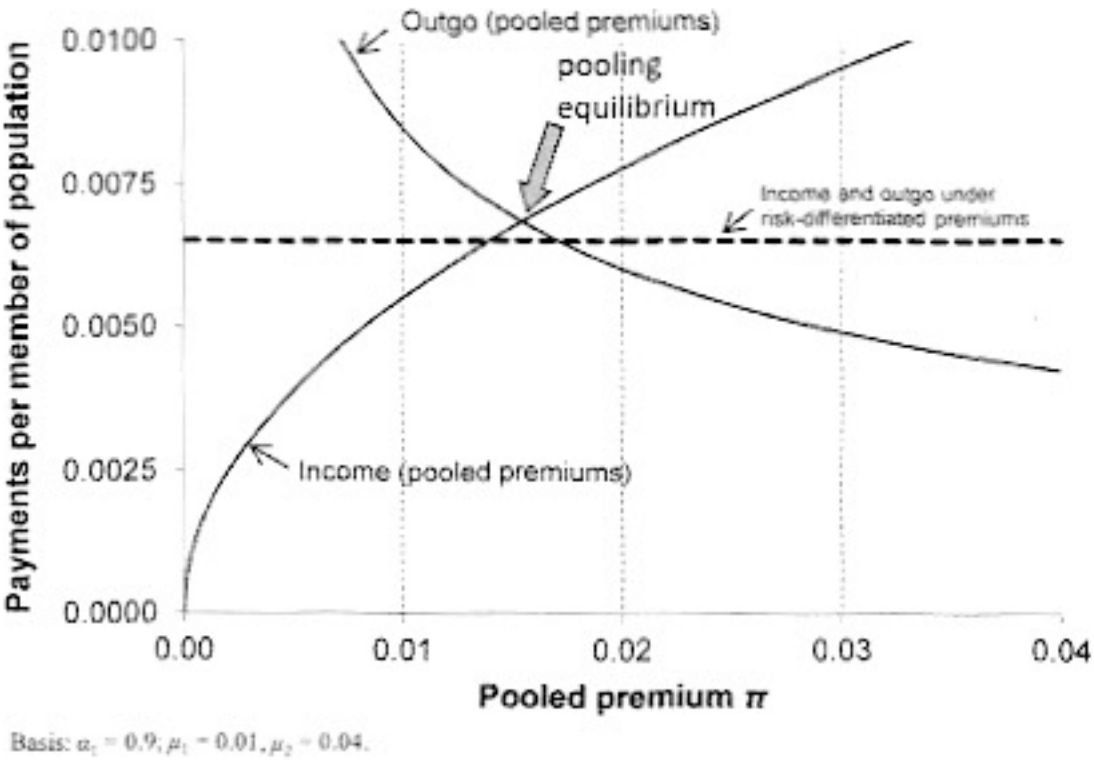
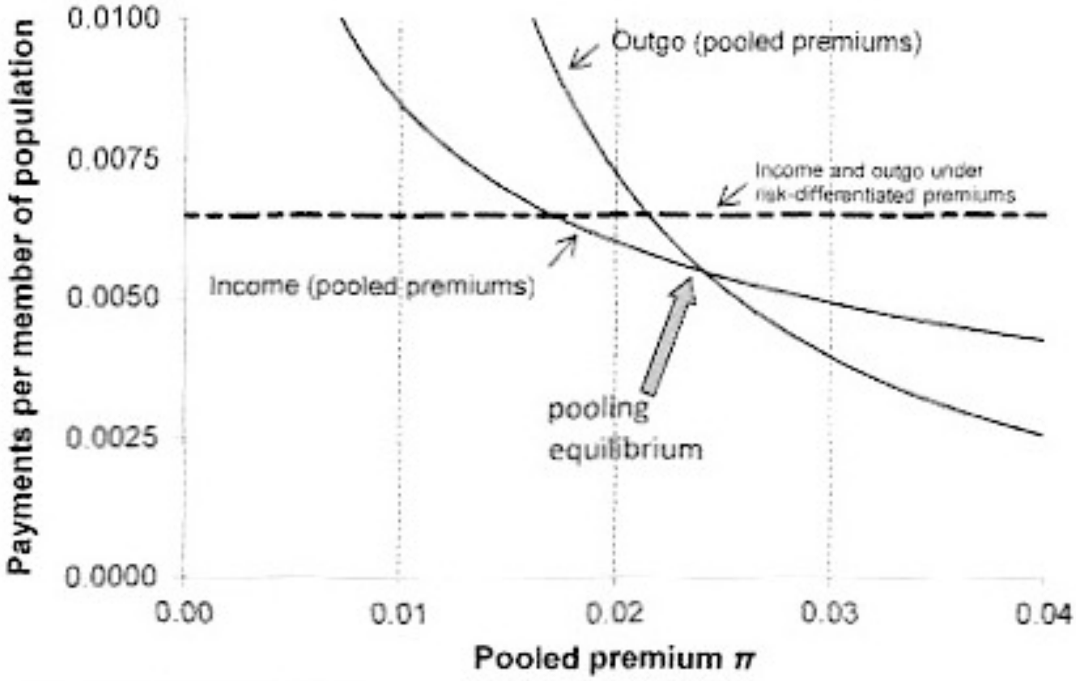


Figure 2.2 shows the equilibrium for relatively inelastic demand, $\lambda = 0.5$.
Figure 2.2 can be interpreted as follows:
The horizontal dashed line is a reference level representing income and outgo if risk-differentiated premiums are charged: that is , if $\pi_1 = \mu_1$, $\pi_2 = \mu_2$ in Equation (2.5) and (2.6).
The curves represent how income and outgo as defined in Equation (2.5) and (2.6) vary with the level of a common ('pooled') premium which is charged to both risk-groups.

On the left-hand side of the graph, where the premium π is low, demand for insurance at this price is high, and so the outgo is high.
Because of the very low premium, income is low (despite the high demand); the markets is far from equilibrium and insurers make large losses.
Insurers will therefore increase the pooled premium, and some customers will leave the market. As customers leave the market, outgo decreases monotonically (the downward sloping curve), but income increases because the number of the customers leaving the market is outweighed by the increasing in premium collected for each remaining customer. So for demand elasticity $\lambda < 1$, the curve of total income slopes upwards. The intersection (shown by the arrow) of the curves for income and outgo represents a pooling equilibrium. The premium at this intersection is the equilibrium pooled premium π_0.
Note that the arrowed intersection is at a higher level of income and dugout than under full risk differentiation. In other words, with the low demand elasticity $\lambda = 0.5$ assumed here, loss coverage under pooling is increased compared with that under risk-differentiated premiums.

Figure 2.3 High elasticity : $\lambda = 1.5$, giving reduced loss coverage.



Basis: $\alpha_1 = 0.9$; $\mu_1 = 0.01$, $\mu_2 = 0.04$.

Figure 2.3 shows the result for more elasticity demand $\lambda = 1.5$, with all other parameter as Figure 5.2. Note that the equilibrium is at a lower than level of income and outgo when risk-differentiated premiums are charged. In other words, the higher demand elasticity $\lambda = 1.5$ assumed here, loss coverage under pooling is reduced (decreased) compared with that under risk-differentiated premiums.

General Results for Iso-elastic Demand with Common Elasticity λ

This section states and illustrates general results for adverse selection, insurance demand (cover) and loss coverage under the iso-elastic demand function as per Equation (2.3) , with a common elasticity parameter λ for both risk-groups. Mathematical proofs are omitted in the book, but have been published in Hao et al. (2015, 2016a).

In interpreting these results, note that a loss coverage ratio (LCR) above 1 (LCR >1) signifies a 'good' outcome from restricting risk classification, and LCR < 1 signifies a 'bad' outcome.

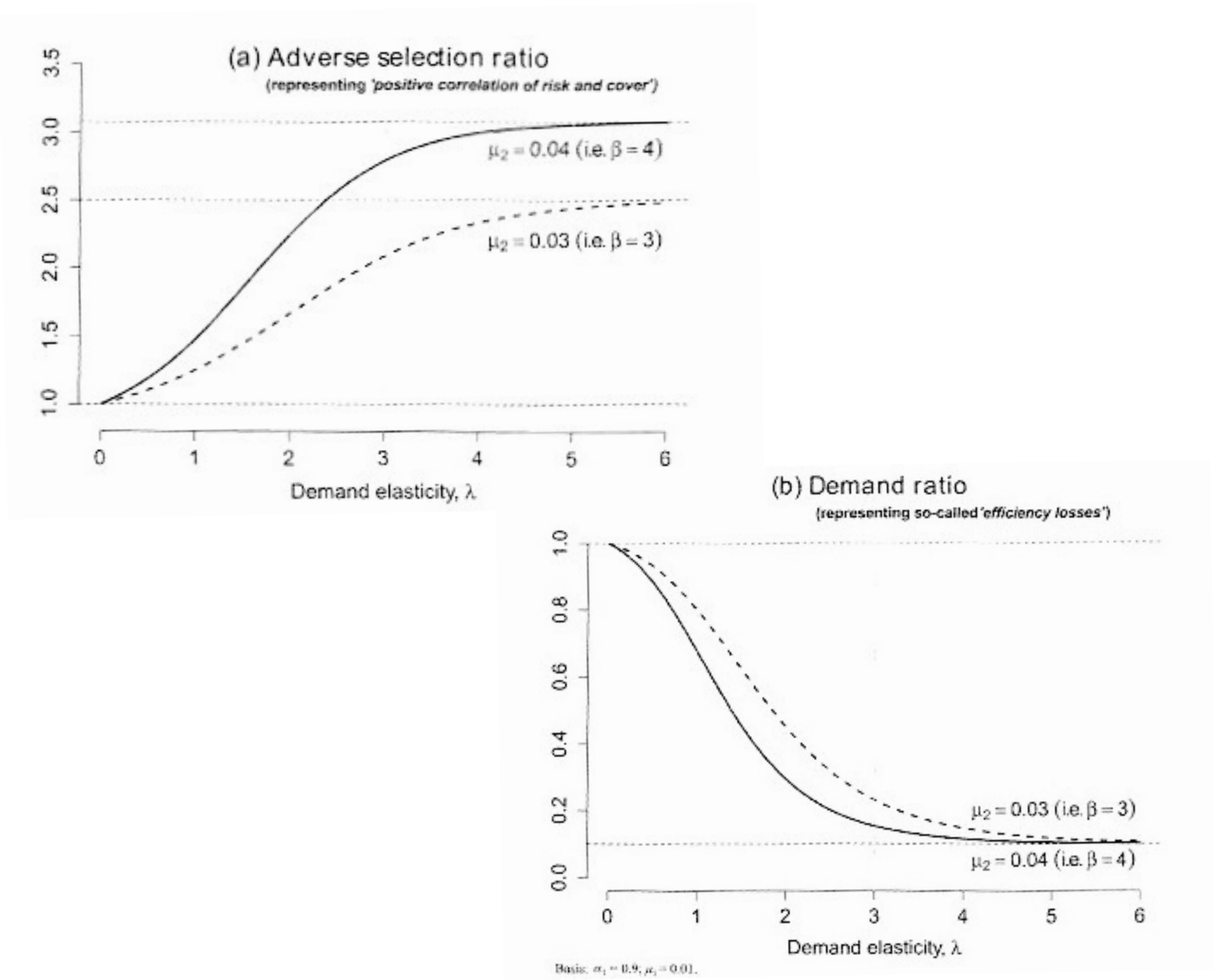


Figure 2.4 : Adverse selection ratio and demand ratio as a function of demand elasticity, for two relative risks.

(a) Adverse Selection ratio increases monotonically with demand elasticity, to an upper limit where the only remaining insureds are high risks. This is shown in upper panel of Figure 2.4 for two different relative risks $\beta = 3$ and $\beta = 4$. Note that the asymptotic limiting values of adverse selection ratio are equivalent to the pooled premium when all lower risks have left the insurance market, divided by the weighed average premium under risk-differentiated premiums.

(b) The corresponding change in demand ratio is shown in the lower panel of Figure 2.4. Recall from the previous part of mathematical details that demand ratio represents insurance demand when risk classification is restricted divided by insurance demand under fully risk-differentiated premiums. Demand ratio is therefore a measure of the reduction in cover which arises from adverse selection, it is realisation in our model of what economic rhetoric typically describes as 'efficiency losses' or 'inefficiency' arising from adverse selection.

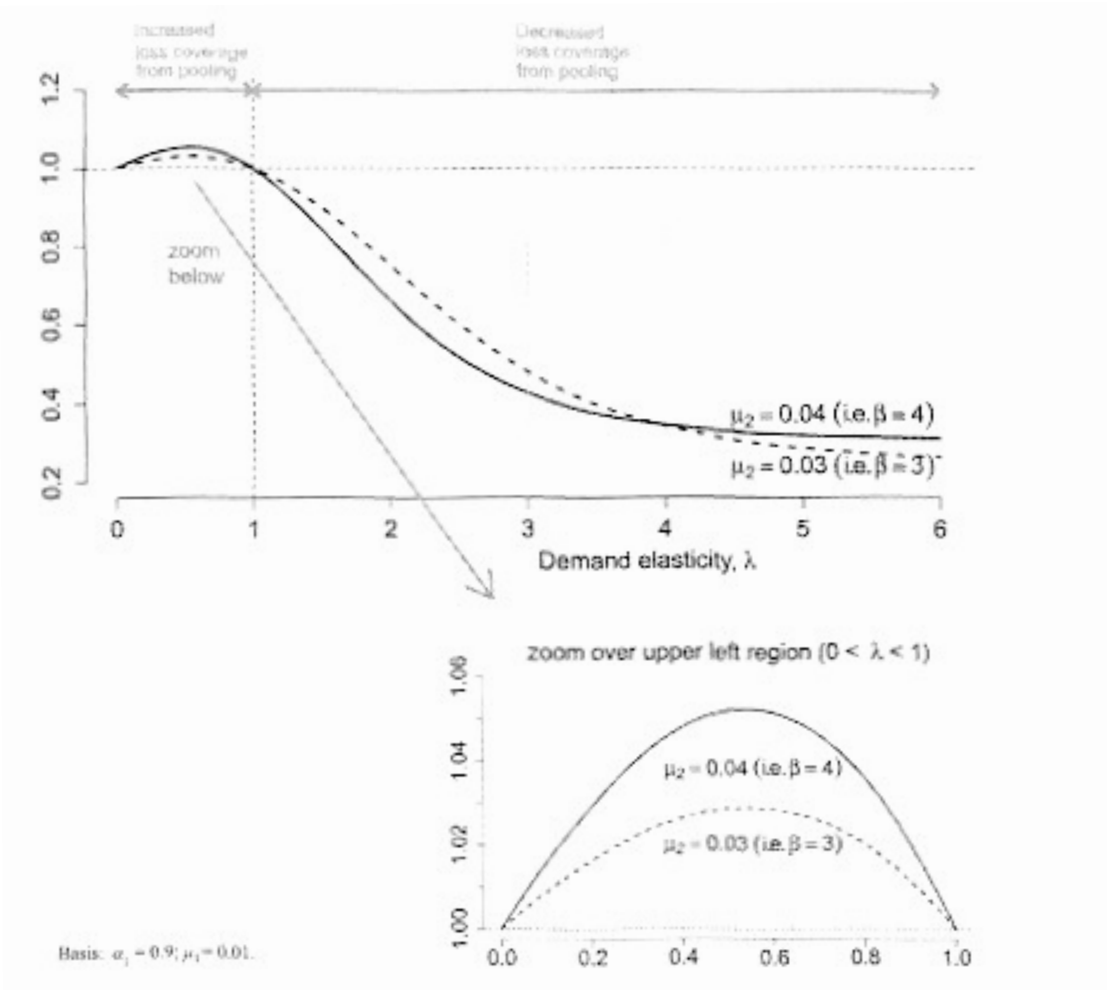


Figure 2.5 Loss Coverage ratio as a function of demand elasticity, for two relative risks.

(c) In contrast to adverse selection ratio and demand ratio, LCR as a function of demand elasticity has an interior maximum, as shown in Figure 2.5.
In other words, loss coverage is maximised with a non zero level of adverse selection, irrespective of whether adverse section is characterised as 'positive correlation of cover and losses' (as in this book, and as in most econometric zests for adverse selection) or as 'reduction in cover' (what economic rhetoric call 'efficiency losses').

(d) For iso-elastic demand, demand elasticity of 1 always gives LCR of 1, as shown in Figure 2.5 (i.e. both curves pass through the coordinate(1,1)).
Lower values of demand elasticity given LCR above 1 and vice versa, that is

$\lambda < 1 \Rightarrow LCR > 1, \lambda > 1 \Rightarrow LCR < 1$, $\lambda = 1 \Rightarrow LCR = 1$.
So for the iso-elastic demand function, $\lambda = 1$ represents a critical value of demand elasticity, which determines whether loss coverage is increased or reduced by pooling all risks in a single class, as compared with loss coverage under risk-differentiated premiums.

(e) For high values of λ, loss coverage flattens out at a lower limit where the only remaining insureds are high risks. This is shown towards the right side of the main graph in Figure 2.5.

(f) The smaller graph on the lower right in Figure 2.5 zooms over the upper left region of the main graph , that is the region where $0 < \lambda < 1$. It can be seen that as demand elasticity increases from zero, LCR increases from 1 to a maximum at around demand elasticity $\lambda = 0.5$. A higher relative risk β gives a higher maximum value of LCR.Note that in this region $0 < \lambda < 1$, the common characterisation of adverse selection as 'inefficient' seems unreasonable. With adverse selection, more risk is being voluntarily traded, and more losses are being compensated.

(g) The value of the maximum for LCR in the zoom region in Figure 2.5 is higher for $\beta = 4$ than for $\beta = 3$. The maximum is also affected by the population structure. To be precise, the maximum depends on the fair-premium risk-share, say w (note: not the same as the fair-premium demand-share α_i , previously defined in Equation (2.4)), i.e. the fraction of total insured risk which lower risks represent when actuarially fair premiums are charged :

$$w = \frac{\alpha_1 \mu_1}{\alpha_1 \mu_1 + \alpha_2 \mu_2} \qquad 2.8$$

(h) Figure 2.6 (see below) shows loss coverage for three population structures, all with relative risk $\beta = 4$. The smaller graph on the lower right zooms over the region $0 < \lambda < 1$. The curve with the highest maximum for LCR is that correlation to population structure $\alpha_1 = 0.8$; this corresponds to a fair-premium risk-share:

$$w = \frac{0.8 \times 0.01}{0.8 \times 0.01 + 0.2 \times 0.04} = 0.5 \qquad 2.9$$

(i) For the iso-elastic demand function used in this chapter, it can be shown that if both dean elasticity and population structure can vary, the maximum value for LCR occurs when $\lambda = 0.5$ and $w = 0.5$.
This maximum value for LCR is :

$$max_{w,\lambda} LCR = \frac{1}{2}(\sqrt[4]{\beta} + \frac{1}{\sqrt[4]{\beta}}) \qquad 2.10$$

(j) The form of Equation 5.10 combined with the requirement for $w = 0.5$ suggests that pooling will be particularly beneficial to loss coverage when there is small risk-group with very high risk (since this combination allows high relative risk β to be combined with fair-premium risk-share $w \approx 0.5$). One obvious example is a small fraction of population with an adverse genetic profile .

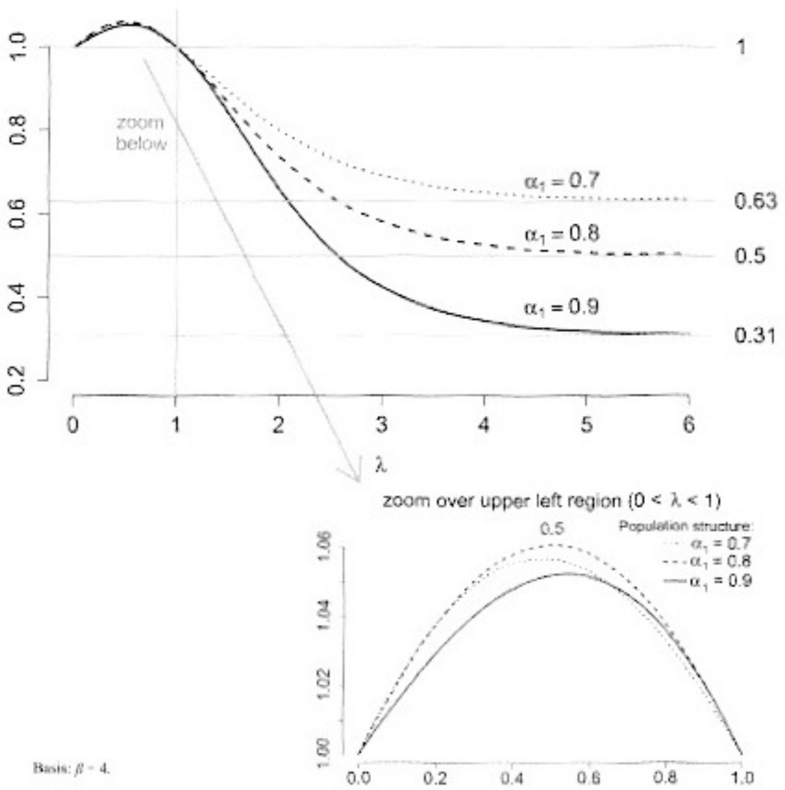


Figure 2.6: Loss coverage ratio as a function of demand elasticity, for three population structures.

General Results for Iso-elastic Demand with Different Elasticities $(\lambda_1 \neq \lambda_2)$

When the elasticity parameter λ_1 and λ_2 for low and high risk-groups are different, it becomes harder to make concise general statements about how they affects loss coverage. The general pattern of results is shown in Figure 2.7 . This shows the regions in the (λ_1, λ_2) plane where LCR is above or below 1 (i.e pooling gives higher or lower loss coverage than risk-differentiated premiums). This graph can be explained as follows.

(a) First, ignore the two dishes curves in Figure 2.7 and focus just on the solid curve for $\beta = 4$ (a relative risk of 4, e.g $\mu_1 = 0.01, \ \ \mu_2 = 0.04$).
This curve demarcates a left-hand unshaded region containing all combinations of (λ_1, λ_2) for which LCR >1 , and a right-hand shaded region containing al combinations of (λ_1, λ_2) for which LCR < 1.
Note that LCR > 1 is associated with moment towards the the upper left of the graph: that is, to give LCR >1, λ_1 needs to be 'sufficiently low' relative to λ_2.

(b) Second, note that λ_1 sufficiently low relative to λ_2 does not necessarily mean $\lambda_1$lower than λ_2. In particular, in the lower part of the unshaded area inside the unit square in Figure 2.7 (the narrow segment below the dashed 45° line, but above the solid curve), λ_1 is slightly higher than λ_2, and yet still 'sufficiently low' to give LCR > 1.

(c) Third, focus now on the two dashed curves in Figure 2.7. The dashed curves illustrate how the solid curve demarcating the left-hand LCR > 1b (unshaded) and

right-hand LCR < 1 (shaded) regions shifts as the relative risk β changes. Note that as relative risk β increases, the curve demarcating the regions becomes more convex, so that a greater range of combinations of λ_1 and λ_2 inside the unit square gives LCR > 1.
But the effect is small: increasing the relative risk from 4 to 40 times or even 400 times makes only as small difference to the curve.

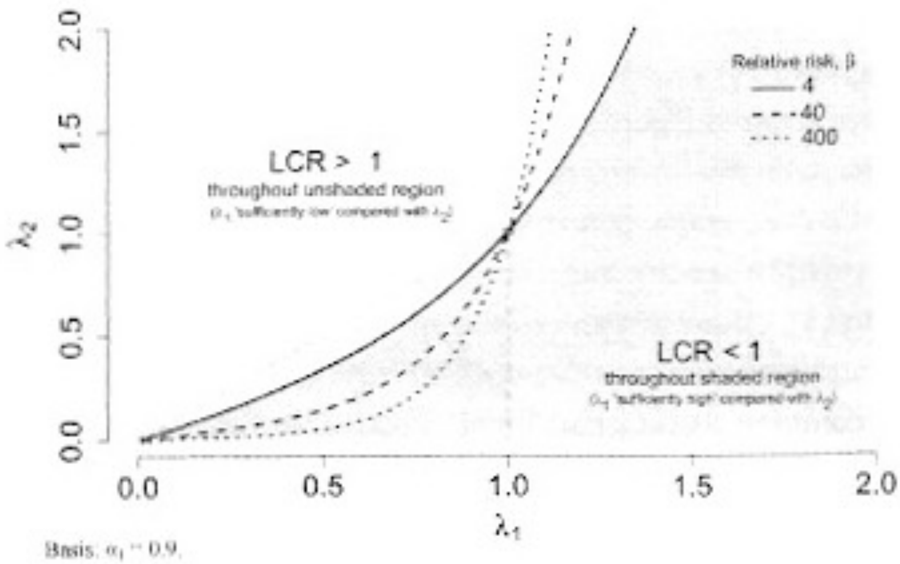


Figure 2.7: Regions of (λ_1, λ_2) space where loss coverage ratio is greater or less than 1.

Comparison with Empirical Demand Elasticities: $\lambda < 1$ is Realistic

The result summarised in Figures 2.4 - 2.7 suggest that under iso-elastic demand , pooling will give higher loss coverage than fully risk-differentiated premiums:

(a) in the equal elasticities case, whenever demand elasticity is less than 1;

(b) in the different elasticities case, whenever $\lambda_1 < 1$ and $\lambda_1 < \lambda_2$ (in Figure 2.7, the part of the unshaded region vertically above the $\lambda_1 = \lambda_2$ diagonal and to the left of $\lambda_1 = 1$);

(c) for some values outside this range, provided λ_1 is 'sufficiently low' relative to λ_2 (in Figure 2.7, other parts of the unshaded region).

How do these limits compare with demand elasticities in the real world? There is some evidence that insurance demand elasticities are less than 1 in many markers. Table 2.1 shows some relevant empirical estimates. (For convenience the elasticity parameter in this part of mathematical details was defined as a positive constant, but estimates in empirical papers are generally given with the negative sign, so the table quotes them in that form.) These estimates suggest that if the iso-elastic demand model is reasonable, loss coverage might often be increased by restricting risk classification.

Table 2.1 Estimates of demand elasticity for various insurance markets

Market and country	Estimated demand elasticities	References
Yearly renewable term life insurance, USA	–0.4 to –0.5	Pauly et al. (2003)
Term life insurance, USA	–0.66	Viswanathan et al. (2007)
Whole life insurance, USA	–0.71 to –0.92	Babbel (1985)
Health insurance, USA	0 to –0.2	Chernew et al. (1997), Blumberg et al. (2001), Buchmueller and Ohri (2006)
Health insurance, Australia	–0.35 to –0.50	Butler (1999)
Farm crop insurance, USA	–0.32 to –0.73	Goodwin (1993)

4 Conclusion

The purpose of this paper is to dispute the conventional thinking of actuaries and economists about the value of adverse selection in insurance pricing markets with the concept of loss coverage. The argument in this paper are based on the book “Loss Coverage : Why Insurance Works Better With Some Adverse Selection”, written by Guy Thomas, which argues that actuaries and economists are overlooking an important point about adverse selection. In the introduction of this paper introduces you to the meaning of adverse selection and some related topics, and the scope and focus of the argument, which leads to the argument. The argument is divided in two parts. The first part is the introduction to the argument, of course this part introduces you to the argument and there is also an easy to understand toy example that tells us that some adverse selection may lead to a beneficial aggregate position for society in terms of increasing the overall level of loss coverage. In the second part there is the argument (with some helpful figures and table) that is built with a mathematical foundation that examine the demand elasticities for both higher and lower-risk groups in different markets and.

Three interesting topics that I haven’t mentioned in this paper but worth knowing:

- Loss Coverage under alternative Demand Functions[3]
- Multiple Equilibrium : A Techical Curiosity[4]
- Partial Risk Classification, Separation and Inclusivity[5]

[3]Loss Coverage: Why Insurance Works Better With Some Adverse Selection, Appendix A, p.245-251

[4] Loss Coverage: Why Insurance Works Better With Some Adverse Selection, Appendix B, p.252-258

[5] Loss Coverage: Why Insurance Works Better With Some Adverse Selection, Partial Risk Classification, Separation and Inclusivity p.89-103

Reference

[1] Thomas, G. (2017). Loss Coverage: Why Insurance Works Better with Some Adverse Selection. Cambridge: Cambridge University Press.
[2] Thomas, G. (2017). Why Insurance Works Better with Some Adverse Selection. available at: https://aheblog.com/2017/06/14/why-insurance-works-better-with-some-adverse-selection/
[3] https://en.wikipedia.org/wiki/Adverse_selection.