Cash Flow Projection Models & Economic Scenario Generators

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March, 2019
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1 Introduction

Nowadays, the insurance industry is extremely science-based, which means it involves a whole process of exchange and trade various forms of risk in a complex and ever-changing social, technological, and competitive environment. Models are important for the managers of insurance’s companies to answer the increasing assessment issues they are facing in the strategic and day-to-day management of their companies. Projecting cash flows is important because it gives the managers a clearer picture of where the business is headed and how they can make improvements. Cash flow projections can help them predict coming cash surpluses or shortages. They can see which periods have more income or expenses. So modelling all comes down to finding the right formulas to reflect as realistically as possible the running of the company.

The following paper is based on the book Modelling in Life Insurance - A Management Perspective (Laurent et al.) and it consists of two parts. In the first part I review the main questions and issues that arise when developing cash flow projection models, in particular I focus on the challenges when it comes to forecasting asset and liability cash-flows under various economic scenarios.

Analyzing economic and financial scenarios is a decisive element for insurers managing long-term risk because of the importance in this context of the investment proceeds from sums placed as reserves. This analysis underlies insurers’ policies towards asset/liability management (ALM) by allowing them to arbitrage between performance and risk for the various possible asset allocations. For life insurers (particularly for savings and retirement), the performance of an asset plays an even greater role in risk management since it contributes to determining the level of liabilities through profit-sharing plans underlying profit uprating policies. In this case, the economic scenarios have a direct impact on commitment levels. So to generate those economic scenarios the use of economic scenario generator is needed which will be discussed in the second part of the paper. Economic scenario generator (ESG) is a software tool that simulates future paths of economies and financial markets, and illuminates the nature of risk elements within the economy that drive financial variability. Emphasis here is placed on highlighting some theoretical and practical focal points related to establishing ESGs in this context, focusing especially on calculating reserves.
2 Cash flow projection model

2.1 Basic concepts

The projection of cash flows is done over a fixed period, which should match either the farthest expected lapse of the last contract of the portfolio (i.e. run-off mode) or a study-specific term (with or without future new businesses). Thus, the model should be flexible enough to produce simulations at different horizons, from short term to long term, depending on the needs they are dedicated to answer. This is not really a modelling issue but it is potentially difficult, if not impossible, to set the model parameters and variables in order to reach the same level of precision and robustness in all cases. A systematic behavioural law (such as, for instance, mechanically selling a percentage of the equity portfolio each year) may be consistent in the long term horizon of a contracts’ liquidation but may give unrealistic results if we focus only on a particular projection year.

A cash flow statement, profit and loss statement and balance sheet are three financial statements a company issues quarterly and annually. A cash flow statement is a financial statement that provides aggregate data regarding all cash inflows a company receives from its ongoing operations and external investment sources, as well as all cash outflows that pay for business activities and investments during a given period. Projected cash flows are balance sheet positions at each observed time step. A balance sheet (Figure 1) it is a financial statement that provides a snapshot of what a company owns (assets) and owes (liabilities), as well as the amount invested by shareholders.

![Figure 1: Assets and liabilities balance sheet](image)

To assess cash flow requires projecting some elements of the profit and loss account as they have impact on assets and liabilities. The profit and loss statement is a financial statement (Figure 2) that summarizes the revenues, costs and expenses incurred during a specified period.
As shown above cash flow model has to simulate the liabilities’ and assets’ cash flows and also interactions between them which means that for the cash flow modelling we actually need three sub-models:

- the liability model
- the asset model
- the asset and liability management model (ALM model).

Both asset and ALM models are to be run in a stochastic mode, which in turn requires having an additional sub-model generating the required scenarios, i.e. the economic scenario generator (ESG) which will be discussed more precisely later on in the paper.

Another thing that needs to be considered is that cash flow modelling depends on the features of the marketed contracts. Therefore there is a structural difference between Savings contracts and Protection contracts. Life-based contracts tend to fall into two major categories:

- Protection policies – designed to provide a benefit, typically a lump sum payment, in the event of a specified occurrence.
- Savings policies – the main objective of these policies is to facilitate the growth of capital by regular or single premiums.

That being said, each topic discussed hereafter will be broken down into those two categories.

2.2 Modelling cash flows from liabilities

**Structure of a Savings model:**

Modelling multi-funds contracts in a unique environment (i.e. global model), enables us to take into account statistical switches between all funds, General Funds (GF) and Unit-linked (UL) alike. But also enables to simulate a more realistic dynamic behaviour of policyholders. So normally the policyholders manage the contracts as a whole making no difference between GF and UL. They surrender their contracts globally, making no
difference between GF and UL. But when they get loans backed by their contracts, the financial hedge is made of GF and UL. Of course this realistic modelling is relevant only if inputs for the model are available at the correct level of granularity. If that is not made so then it can cause problems such as could ruin its expected benefits and, as a consequence, drastically change the costs/rewards balance expected when deciding to develop it. Modelling switches between all funds of a Savings multi-funds contracts also implies modelling switch fees levied on contracts and switch commissions paid to commercial networks.

The choice of a number of modelled funds it is also an important issue. First, the number of modelled funds defines the granularity of information required to run the model. Second, the higher the number of modelled funds, the longer and the costlier is the processing by the model. The model will demand more and more machine power in a proportion that may be exponential to the number of modelled funds. This capability of running the model in a reasonable timeframe and at a reasonable cost is of big importance, taking into consideration the average huge number of investment funds marketed in multi-funds contracts (more than one thousand) and notably, in the context of a very short reporting process.

Structure of a Protection model:
Modelling in a unique environment allows modelling the interactions between risks more or less precisely. In its rougher version, the multi-risk environment focuses on ensuring that once an insured is dead, he is no longer considered to be in another risk status (disabled or unemployed). In its more sophisticated version, the multi-risk environment allows us to manage the transition, policyholder by policyholder, from one health status to another: for instance, from a sound health state to a degrading health state (temporary or permanent disability).

The choice of a structure has consequences on how the models will be fed. The more sophisticated the structure, the higher the volume of the data. In addition to the issue of getting these data is the issue of ensuring that they are at the required quality level.

Modelling cash flows from insurance contracts:
It all starts with inventorying transactions on contracts (collecting premium, surrender payment, claim payment, contract cancellation . . .) because they are the basic business-related cash flows reflecting decisions made by policyholders. Some issues on these basic cash flows deserve to be briefly commented on.

On Protection perimeter, claims’ modelling should distinguish according to the timeline: claims that have been incurred before the projection date—called “in-force” claims, and after the projection date—called “future new” claims. Claims’ modelling is not unknown to the Solvency I framework. Indeed, if a large part of the claims reserves is constituted by the reported claims assessed file by file, the balance is composed of unknown incurred claims because they are reported late to the insurer. To evaluate these IBNR (incurred but not reported claims), the insurer uses simulation techniques. Different modelling methods for claims are then possible depending on the date the calculation is done.

In addition to the basic insurance contracts’ cash flows, the model designer should also model the setting-up of the technical reserves. This stage is quite significant as the change
in value of technical reserves is instrumental to the final value of the Profit&Loss statement.

In particular, for the Savings contracts, the modelling of the change in value of the mathematical reserves (MR) comes down to scheduling the management acts (i.e. the basic cash flows: premium collection, claim payment, switches, etc.) whose features have been previously defined. The point here is to position them consistently on the timeline based on agreed conventions. For instance, monthly deaths all occur at the beginning of the month, so that no monthly premium is collected. This consistent scheduling is quite time consuming in the development phase. Always on Savings perimeter, a key element in the change in value of MR is their revaluation at the credited rate, i.e. the contract’s annual return rate that is made of both the minimum guaranteed rate (MGR), if any, and the policyholders’ profit sharing. The modelling of the profit sharing is addressed in the following section relating to assets and liabilities interactions. The modelling of the MGR revaluation should cover different situations: one-year MGR, multi-year MGR, variable MGR. The final number of modelled MGRs is decided by the level of aggregation that produces the results at the right level of analysis, as defined by Top Management.

**Modelling cash flows from overheads:**
Overhead expenses are all costs on the income statement except for direct labour, direct materials, and direct expenses. Nowadays, overheads are closely monitored and, often, subject to strong action of the Top Management. Some issues have to be addressed: First issue is the scope of overheads to be projected. In the run-off portfolio simulation, the administration costs are those to be closely considered, as the acquisition costs would intervene only marginally. Within administration costs, it is also necessary to isolate the exceptional non-recurring overheads whose projection will give erroneous results. As the definition of exceptional non-recurring overheads might be loose or differ from one company to the other, it is very important to document the reasons supporting their qualification as exceptional and non-recurring.

The projection term is another decision to be made that has an impact on the overheads’ issue. The farther the projection term, the more significant is the impact of the overheads projection on the modelled results.

**2.3 Modelling cash flows from assets**
Assets’ cash flows represent cash flows generated by the financial instruments existing in the insurer’s investment portfolio at the simulation date. Assets’ cash flows are assessed using the ESG in order to take into account possible financial markets’ fluctuations in the future.

The first step in assets’ cash flow modelling is to set asset classes. To decide the right scope of asset classes, one should begin by looking at the asset mapping as shown by the detailed investment reporting. A homogeneous asset class (Figure 3) regroups assets that have the same cash flow pattern and the same risk profile. In other words, they could be modelled with the same formulas.

The second step of the modelling is to model the investment strategy, i.e. the interactions between assets, because in the valuation process insurer’s assets should not be considered
as independent from one another or from the global running of the company.
Most of the time, assets should respect an asset allocation, called an “asset mix”, which
indicates a target (in proportion to the total asset value) for each asset class. The reval-
uation of a given class then implies quite automatically whether to sell or to buy some
assets of the other classes, otherwise the allocation objectives may not be respected.
For instance, if the strategic asset allocation is to maintain the initial level of bonds, when
interest rates are increasing, the market values of the bonds are reduced by the applica-
tion of the bonds’ closed formula. Therefore the bond asset class is weighted less in the
company’s asset portfolio and, in some cases, this weight might be out of the authorized
range as set by the company “asset mix”. To return to the target mix, some bonds need
to be bought which, in turn, means selling other assets in the portfolio in order to get
the cash to do so, via equities selling, for example.
Many other possible situations need to be taken into account in the cash flow projection
model and require rules and benchmarks to be modelled in the most realistic possible
manner, in line with true asset management.

<table>
<thead>
<tr>
<th>Assets mapping</th>
<th>Assets classes</th>
<th>Assets modeling type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equities (CAC40, S&amp;P500)</td>
<td>Equities</td>
<td>Equity modeling type</td>
</tr>
<tr>
<td>Physical Real Estate</td>
<td>Real Estate</td>
<td>Equity modeling type</td>
</tr>
<tr>
<td>Fixed rate government bonds</td>
<td>Fixed rate bonds</td>
<td>Bonds modeling type</td>
</tr>
<tr>
<td>Variable rate government bonds</td>
<td>Variable rate bonds</td>
<td>Bonds modeling type</td>
</tr>
<tr>
<td>Inflation rate government bonds</td>
<td>Indexed bonds</td>
<td>Bonds modeling type</td>
</tr>
<tr>
<td>Barrier caps</td>
<td>Rate options</td>
<td>Rate option modeling type</td>
</tr>
<tr>
<td>Alternatives, hybrids</td>
<td>Structured assets</td>
<td>( X% \times \text{Equity modeling type} ) ((1-X%) \times \text{Bond modeling type})</td>
</tr>
<tr>
<td>Equities Unit Linked</td>
<td>Unit Linked</td>
<td>Equity modeling type</td>
</tr>
</tbody>
</table>

Figure 3: Example of asset class grouping

2.4 Assets and liabilities interactions modelling

2.4.1 Assets and Savings Liabilities Interactions

The financial strategy:¹

These ALM interactions together with the investment strategy form what we call the
“financial strategy”. This strategy explains movements from the initial balance sheet to

¹Laurent: Modelling in Life Insurance - A Management Perspective, p.76
the final balance, as displayed hereafter (Figure 4).
From the initial balance sheet, the first movement is the yearly cash inflows and outflows from recurrent operations, both the asset cash flows (dividends for equities, coupons and maturities for bonds...), and the liabilities cash flows (premiums minus benefits), generating a result.
The second movement comes from the financial markets’ fluctuation over the period and their impacts on the purchases and sales of assets made in order to be in the asset mix range. This movement is made of two components: the first is the change in assets’ values due to the application of economical scenarios; the second is the implementation of the investment strategy.
The third movement is both the calculation and allocation of profit sharing to the mathematical reserves. The calculation depends on all or a proportion of the results generated by the two first movements.
The fourth movement is the assets and liabilities realignment which consists in selling assets (equities, bonds, and cash) proportionally to their respective weight in the total asset amount in order to make assets match with liabilities. Doing so, the assets sold in excess of liabilities crystallize the current period result.
This financial strategy and its sequence of movements are repeated for each economical scenario, and for each calculation time period up to the term of the projection horizon. This means that for a stochastic run made of one thousand economical scenarios and for a forty year horizon, the financial strategy runs forty thousand times.

Figure 4: The financial strategy
Policyholders’ Profit Sharing Modelling:

The target profit sharing is the value that the company is willing to give to policyholders at each simulated period in addition to the minimum guaranteed rate in order to be consistent with its participation policy. This target rate depends on the asset yield. It also reflects Management’s anticipation of the competitors’ credited rate. It is consistent with:

- The wealth position of the Company at the end of the period, notably the unrealized gains or losses on assets,
- The credited rate of the previous year,
- The financial markets’ situation,
- The top management’s will to strategically promote some contracts.

In terms of modelling, the credited rate may be deduced from a formula decided at the start of the projection period. This formula is often unchanged during the projection period. This is mainly because changing it would have a significant negative impact on the calculation runtime, despite it having a positive impact on the results. Therefore, the modelling choice for the implemented formula is very important as you will have to stick with it over the tens of projected years. Making an error will have a direct impact on the indicators, such as the liabilities fair value or the value of future profits. For instance, a very high target profit sharing will trigger all levers modelled in order to reach it whatever the financial markets’ situation: the insurer’s profit will be quite low in these circumstances. In addition, this very high target will over the long projection horizon generate surrenders and switches with a high level of probability.

2.4.2 Assets and Protection Liabilities Interactions

There are many such interactions, one of them is the interaction between creditor payment insurance contracts (which is a type of life insurance policy purchased by a borrower that pays off one or more existing debts in the event of a death and the underlying mortgage contracts): actual interest rates may push borrowers to reimburse their loans early through renegotiation. This in turn would result in early insurance policy surrenders. Therefore the question may be asked as to whether modelling these dynamic surrenders or not.

3 Economic Scenario Generator

The projection of economic and financial risk factors is a key element of prospective analyses made by life insurers, both for the calculation of reserves under Solvency 2 and for the asset allocation and management of financial risks. This projection is achieved in practice through “economic scenario generators” which are inputs for the calculus of the economic value of assets and liabilities and the analysis of the distribution of this value.

In order to express the need of an ESG, we will take a Life Profit Sharing product as an example. The policyholder’s premium is invested in a pool of assets. Then, he will
receive the maximum between a share of the profit and a Minimum Guaranteed Rate. Those products are linked to the underlying assets’ performance and the policyholder’s behavior should be taken into account because he can surrender from the contract at any time.

Because the Cash Flows are dependent of the underlying risk, the Best Estimate (BE) can be expressed as follows:

\[ BE_{stochastic} = \mathbb{E}\left[ \sum_{t \geq 1} \delta(t) CF(t) \right] = \mathbb{E}\left[ \sum_{t \geq 1} \delta^{(k)}(t) \mathbb{E}[C F(t) | R^{(k)}] \right] \]

Where:

- \( \delta^{(k)}(t) \): Discount rate for the \( k^{th} \) scenario at time \( t \);
- \( CF(t) \): Cash Flow at time \( t \);
- \( R^{(k)} \): Here, the Cash Flows depend on the financial risks \( R^{(k)} \) (Assets valuation and Interest rates).

Then, we use the different scenarios produced by an ESG to compute the BE with Monte-Carlo simulations:

\[ BE_{stochastic} = \frac{1}{N} \sum_{k=1}^{N} \sum_{t \geq 1} \delta^{(k)}(t) \mathbb{E}[C F(t) | R^{(k)}] \]

If we consider that the underlying products assets are:

- Bonds: We will need Interest rates scenarios;
- Equities: We will need Equities returns scenarios.

Here, we see the usefulness of using several scenarios. In fact, it will trigger specific dynamic behavior of the policyholder in the case the asset or interest rate fall down.

In this example, we saw where ESG scenarios could be useful. They are used to produce scenarios according several financial and economic quantities among which: interest rates, equities returns, inflation rate, credit spreads... Those quantities are simulated using mathematical models (Black-Scholes model, Vasicek Model...).

We will limit ourselves here to explicit financial risks in that they are directly linked to traded asset prices on the market. In this restrictive framework, the relationship between the risk factors and the price of assets may be more or less direct:

- For equities and real estate, the modelled factor is directly the price of the asset.
- For bonds, in general, these are modelled using a limited number of explanatory factors, typically short-term rates. The asset is derivative.

How to project risk factors depends on the use that will be made of the ESG and lead to define the concepts historical probability (for distribution purposes) and risk-neutral probability (for pricing purposes).
3.1 Calculation of reserves

The calculation of reserves for the needs of liquidity (that mostly involve a financial hazard) is homogeneous to a price calculation. Once reserves have been calculated, a one-year balance sheet is forecast to assess the minimum level of required capital to ensure solvency with a probability of at least 99.5%.

An ALM model in life insurance should then be able:
- to compute prices (assets and liabilities);
- to compute quantiles of the distribution of the net asset value, that is prices distributions.

The first item uses risk neutral measure, the second one uses historical measure.

As part of a comprehensive modelling which aims to provide distributions of economic value, we use a two-level approach:

• building a functional $g$ that provides the price vector as a function of the status variables $Y$ at the time of calculation, $\pi_0 = g(Y_0)$ and
• building dynamic model for the risk factors, $Y_t$

We can then determine prices for any date via $\pi_t = g(Y_t)$.

The construction of the functional $g$ is based on the classical assumptions of financial markets including the “no arbitrage assumption”, which leads to construct ”risk neutral” probability that make the discounted prices processes martingales. The construction of the dynamics of $Y$ is a problem of econometrics.

For example, within a Vasicek type mono rate factorial model, we have the following short rates:

• Quantiles: $dY_t = dr_t = a(b - r_t)dt + \sigma dW_t$, $W_t$ being a Brownian motion

• Pricing: $dr_t = a(b_\lambda - r_t)dt + \sigma dW_t^Q$ with $b_\lambda = b - \frac{\lambda \sigma}{a}$ and $W_t^Q = W_t + \lambda \times t$ being a Brownian motion under the probability of $Q$, which allows us to construct the pricing function:

$$g(r_t) = P(r_t, T - t) = \exp\left(\frac{1 - e^{-a(T-t)}}{a}(r_\infty - r_t) - (T - t)r_\infty - \frac{\sigma^2}{4a^3}(1 - e^{-a(T-t)})^2\right)$$

with $r_\infty = b_\lambda - \frac{\sigma^2}{2a^2}$

Building the functional $g$ relies on the conventional assumptions of market finance and in particular the absence of arbitration leading to “risk neutral” probabilities, which make the process of discounted prices a Martingale. Constructing the dynamics of $Y$ is an econometric problem. Finally, we can see that, in this theoretical framework, the two representations are linked via the “market price of risk”, $\lambda$. Note: The parameter $\sigma$ is theoretically invariant.

With financial products, generally, cash flow is expressed in a relatively simple way by using prices of different assets. For example, the cash generated from a European put

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option having a strike price of $K$ on an underlying $S$ is of the form $F(T) = |K - S_T |^+$. Adapting this approach to an insurance context, and more particularly to savings and retirement portfolios, runs up against the complexity of describing the cash flows of the contract. These result, in fact, in complex interactions between the market returns of the assets backing the commitments, accounting rules, the applied rate and profit sharing reserves set by the insurer and policyholder decisions in terms of redemption. Faced with this difficulty, specialists developed a modelling framework summarized by the following diagram:

\[
R = \mathbb{E}^{P_A \otimes Q^F} \left( \sum_{j \geq 0} \frac{F_j}{(1 + R_j)^j} \right)
\]

\[
\approx \frac{1}{N} \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{a=1}^{A} \frac{\text{Cashflows}_{t,n,a} - \text{Premiums}_{t,n,a} + \text{Management fees}_{t,n,a} - \text{Loading}_{t,n,a}}{(1 + R_n(0,t))^t}
\]

The ESG, therefore, feeds a cash flow forecasting model and the amount of reserves $R$ is reached by simulation (Figure 5). In order to be consistent with market values, the financial risk factors entered into the ESG must be modelled under a “risk neutral” probability.

Figure 5: The role of the ESG in an asset/liability model for calculation reserves

The ESG must take into account all the financial risks faced by the insurer or at least the risk-free rate, the credit risk, inflation, stock prices and real estate. While financial derivatives are often built around a reduced number of risks (the loss in value of an underlying, or a counterparty default, etc.), this involves considering them together much more comprehensively. Among these factors, the choice of the rate model is a key element of a “risk neutral” ESG. The model’s ability to adequately represent the price of interest rate derivatives is a criterion that respects consistency with market values. But when examining the choice of market finance models, we see that there is no relevant model for all of the fixed income products and that the model is chosen and calibrated according to the product’s nature; different models are used for CAPS, Swaptions and CMS. The model is chosen and calibrated to best represent the price of the instrument for which it
is used, without claiming to properly represent the prices of other instruments of different structure. Applying this approach in the context of making a best estimate calculation of reserves for savings contracts in euros, implies, a priori, having the prices of reset and cyclical redemption options; information that does not exist and the calculation of which is therefore made in a mark-to-model framework without any direct observable data. We must then rely on surrogate data and use observable prices of interest rate products, which can reasonably be expected to behave like priced options. This choice is somewhat arbitrary and this observation reinforces the normative character of reserve calculations. In other words, if the model set by the regulator is without constraints, the latitudes of choice are significant and deciding whether a model is relevant or not can only be based on criteria of financial theory.

3.2 Coherence

The calculation of economic values leads to model the risk factors in a risk neutral probability, while the analysis of the distribution of these values requires the projection of these factors under the historical probability. Therefore, the insurer must handle different representations of the risk factors, which requires looking at the characteristics of a risk neutral ESG, those of an “historical” one and the possible need for coherence between these two representations.

The issue of coherence also exists between different representations of the same risk factor in different parts of the model. Projection models are indeed rather complex, which may involve various sub-models for the same risk factor depending on the calculation being made. In this context it is important to ensure coherence between these various representations. Calibration is thus determined to minimize the differences between these various curves.

3.3 ESG Process

When using an ESG, the following steps and questions should be asked:

- **Projection context:** Does the projection need to be Risk-Neutral or Real-World? The Real-World approach will try to fit the historical and statistical behaviors of the desired economic quantities. This could be used in an ORSA model to project Business plan assumptions or to optimize the strategic asset allocation. The Risk-Neutral approach is used to simulate scenarios that are consistent with market prices at the calibration date. Those are used in the Solvency II BE calculations.

- **Model choice:** It will depend on the financial quantity projected and the projection context. The model complexity should be taken into account in order to understand the parameters behind the model projection.

- **Calibration:** Crucial step that could lead to misleading scenarios. Users of ESG models need to incorporate a view of future market dynamics into their risk-modeling environment. The process of reflecting these views into an ESG is referred to as model calibration. More specifically, calibration is the process of setting the parameters of
the equations within an ESG model to produce the distributions and dynamics (e.g., volatility, correlations, tail characteristics) of economic and financial variables required by the application for which they are being used. The calibration process is considered as crucial because it will return the model’s parameters used in the diffusion process. A mislead in the data preparation or optimization algorithm could lead to unusable scenarios even if the model is adapted.

- **Simulation**: The stochastic differential equation is discretized in order to project the financial or economic quantity.

Moreover, quantitative and qualitative validations should be done all along the process. Validation ensures that the estimation of an ESG’s parameters results in simulated behavior that is a good representation of the variable or market under consideration. Effective validation of an ESG requires comparing simulated output data with some predefined benchmark of acceptance criteria. For a typical insurance or pension undertaking, the list of financial and economic variables that may be of interest is typically quite large. For this reason, the validation system and validation environment require careful design at inception, in order to organize the various data elements in an ordered fashion.

![ESG Process Diagram](image.png)

Figure 6: ESG Process
3.4 Implementation of the model

Let's consider a unit-linked contract with an underlying asset modelled by a log-normal process and the risk-free rates modelled by a Vasicek model. Vasicek model describes the movement of an interest rate as a factor composed of market risk, time, and equilibrium value, where the rate tends to revert towards the mean of those factors over time. Essentially, it predicts where interest rates will end up at the end of a given period of time, given current market volatility, the long-run mean interest rate value, and a given market risk factor. The Vasicek interest rate model values the instantaneous interest rate using the following equation:

\[
\frac{dr(t)}{dt} = k(\theta - r(t))dt + \sigma dB(t)
\]

Where:

- \(B(t)\) is the random market risk (represented by a Brownian motion); \(t\) represents the time period;
- \(k(\theta - r(t))\) represents the expected change in the interest rate at time \(t\) (the drift factor);
- \(k\) is the speed of the reversion to the mean;
- \(\theta\) is the long-term level of the mean; and
- \(\sigma\) is the volatility at time \(t\).

Price: 
\[
S(t) = S_0 \exp\left(\int_0^t (r(u) - \frac{\sigma^2}{2})du + \sigma B(t)\right)
\]

The contract duration is 10 years, fully redeemed at maturity. In the meantime, the structural redemption rate is 2% and cyclical redemptions, of up to 5%, are added when the value of the share at time \(t\) is lower than the initial value. The best estimate value of the contract is simply the initial value of the share.

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3Laurent: Modelling in Life Insurance - A Management Perspective, p.90,91
From the Figure 7 above we can see that the convergence of an empirical best estimate towards its theoretical value is slow and, after 1000 runs, a gap of about 1.5% still remains. Consistent with the orders of magnitude observed in a life insurer’s balance sheet, Convergence gap depending on the number of simulation runs and under the assumption that the ratio between equity and technical reserves is from 1 to 10, a gap of 1.5% on reserves led to a difference of about 15% on equity.

Furthermore, it should be noted that this is no longer just a sampling error, but a systematic bias that slowly diminishes with the number of runs. Thus in the example above, we see that the actual value of a best estimate is systematically underestimated.

As another Example lets simulate the price calculation of a zero-coupon bond within a mono-factor model. To be even more specific, we set it within the framework of the Vasicek model based on the short rate dynamic \( dr_t = a(b - r_t)dt + \sigma dW_t^Q \), where \( W_t^Q \) which is a Brownian motion under probability \( Q \).

The estimation error associated with simulation-based calculation of zero coupon prices, in this context, would look like this (Figure 8):
Errors logically increase with maturity and decrease with the number of simulation runs.

It can be said that approximating the price of a single zero-coupon through simulation is not easy. After 2000 runs, which constitute the number of simulations for a best estimate calculation, the price of a 30 year zero-coupon is estimated with a relative error of approximately 30 %, which decreases to 10 % after 10,000 simulations. However, the practical consequences of this example should be qualified because if the relative error is significant, the absolute value of a long-term zero-coupon is low and the impact on a best estimate, which combines cash flows from different maturities, is rather small.
4 Conclusion

When seeking to find an answer to the different issues discussed in the first part of the paper, the temptation is to design a dream model with the following features: a robust tool, tested and validated internally and externally, built to mirror the business-model of the company, consistent with different norms and standards, well documented, simple to feed in, giving rich and exhaustive information on outputs, as fast as possible to run, available everywhere in the whole company for multi-users needs, embedded in a solid IT environment, with a high evolution potential and finally at a reasonable cost! This dream model is... a dream. However, it could be approached provided some conditions are fulfilled: Data and parameters are of an appropriate quality; the model is consistent with both the reality of the business and the management rules; the runtime is fast enough to meet the different deadlines.

In the recent years, Solvency II requires insurers the ability to estimate financial and insurance risks. New quantitative measurements have been introduced in order to assess those risks. Moreover, stress tests and adverse scenarios should be taken into account to measure the risks’ volatility. For those reasons, Economic Scenario Generators (ESG) have become more and more inevitable especially for Life insurers. The purpose of the second part of the paper is not so much to locate an ESG that accurately reflects the behaviour of modelled assets, but rather to provide a simple, coherent explainable framework that allows fuelling asset/liability models with credible scenarios shared widely by the market. A goal could be set to establish a number of minimum conditions that an ESG should respect, i.e. that however imperfect it may be, it should be robust enough to ensure its users consistent and comparable economic scenarios, which report financial risks with reasonable effectiveness. As we reach the end of this chapter, it appears that putting together one’s own ESG is within the scope of every organization. Building one’s own scenarios is key to a genuine mastery of risk management by imposing an a priori reflection on risk exposure, their modelling and the consequences of choices made at this level.
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