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PATHWISE LARGE DEVIATIONS FOR THE ROUGH BERGOMI MODEL: CORRIGENDUM‡

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Abstract

This note corrects an error in the definition of the rate function in Jacquier, Pakkanen, and Stone (2018) and slightly simplifies some proofs.

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1. Corrected rate function

Note that the correct rate function also appears in the PhD thesis [3] (see Proposition 1.4.18), but with a different proof. We first give a slightly simplified proof of [1, Theorem 3.1]. Any unexplained notation is as in [1].

Let $Y := \int_0^{\cdot} \varphi(u, \cdot)\,\mathrm{d}W_u$ be the Gaussian process from that theorem, and $K_Y : \mathcal{C}^* \to \mathcal{C}$ its covariance operator (definition in [2, p. 5]). As noted in [1], $\mathcal{I}^\varphi$ is injective by Titchmarsh's convolution theorem. By the factorization theorem [2, Theorem 4.1] and the discussion in [2, pp. 32–33], it suffices to verify the factorization identity $\mathcal{I}^\varphi(\mathcal{I}^\varphi)^* = K_Y$ to conclude that the reproducing kernel Hilbert space (RKHS) is the image $\mathcal{I}^\varphi\big(L^2([0, 1])\big)$. By Fubini's theorem, we have $(\mathcal{I}^\varphi)^*\mu = \int_{\cdot}^1 \varphi(\cdot, t)\mu(\mathrm{d}t)$ for any measure $\mu \in \mathcal{C}^*$. We then compute, for $\mu, \nu \in \mathcal{C}^*$,

$$\begin{aligned}\mu\big(\mathcal{I}^\varphi(\mathcal{I}^\varphi)^*\nu\big) &= \int_0^1 \int_0^t \varphi(u, t) \int_u^1 \varphi(u, s)\,\nu(\mathrm{d}s)\,\mathrm{d}u\,\mu(\mathrm{d}t)\\ &= \int_0^1 \int_0^1 \int_0^{s\wedge t} \varphi(u, t)\varphi(u, s)\,\mathrm{d}u\,\nu(\mathrm{d}s)\,\mu(\mathrm{d}t)\\ &= \int_0^1 \int_0^1 \mathbb{E}[Y_t Y_s]\,\nu(\mathrm{d}s)\,\mu(\mathrm{d}t) = \mathbb{E}[\mu(Y)\nu(Y)],\end{aligned}$$

which proves the theorem.

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The second definition in [1, (2.3)] should be replaced by the following one.

Definition 1. For $\Phi:\mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^{2\times 2}$, define $\mathcal{I}^\Phi:L^2([0, 1], \mathbb{R}^2) \to L^2([0, 1], \mathbb{R}^2)$ by

$$\mathcal{I}^\Phi f:= \int_0^{\cdot} \Phi(u, \cdot)f(u)\,\mathrm{d}u.$$

The following theorem replaces [1, Theorem 3.2].

Theorem 1. *Let φ_1, φ_2 satisfy [1, Assumption 3.1], and define $Y_i:= \int_0^{\cdot} \varphi_i(u, \cdot)\,\mathrm{d}W_u^i$, $i = 1, 2$, where W^1 and W^2 are standard Brownian motions with correlation parameter $\rho \in (-1, 1)$. Then, the RKHS of (Y_1, Y_2) is $\mathcal{H}^\Phi := \{\mathcal{I}^\Phi f : f \in L^2([0, 1], \mathbb{R}^2)\}$, with inner product $\langle \mathcal{I}^\Phi f, \mathcal{I}^\Phi g\rangle = \langle f, g\rangle$, where*

$$\Phi = \begin{pmatrix} \varphi_1 & 0 \\ \rho\varphi_2 & \sqrt{1-\rho^2}\varphi_2 \end{pmatrix}.$$

Proof. Analogous to the proof above. Injectiveness of $\mathcal{I}^\Phi$ follows from the Titchmarsh convolution theorem. We have $(\mathcal{I}^\Phi)^*\mu = \int_{\cdot}^1 \Phi^\top(\cdot, t)\mu(\mathrm{d}t)$ for any measure $\mu \in (\mathcal{C}^2)^*$. The factorization identity $\mathcal{I}^\Phi(\mathcal{I}^\Phi)^* = K_{Y_1,Y_2}$ is verified as above.

Theorem 1 implies the following corollary, which replaces [1, Corollary 3.2].

Corollary 1. *The RKHS of the measure induced on $\mathcal{C}^2$ by the process (Z,B) is $\mathcal{H}^\Psi$, where*

$$\Psi = \begin{pmatrix} K_\alpha & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix}.$$

Consequently, $\|\cdot\|_{\mathcal{H}^\Psi}$ should replace $\|\cdot\|_{\mathcal{H}_\rho^{K_\alpha}}$ in line 4 of p. 1083 and in the proof of [1, Theorem 2.1] on p. 1088. The special case $\rho = 0$ requires no separate treatment, and the result agrees with [1, Section 5].

2. Minor corrections

1. On p. 1079, last line of the introduction: replace $\int_0^1$ by $\int_0^{\cdot}$.
2. On p. 1084, definition of topological dual: add 'continuous' before 'linear functionals'.
3. On p. 1085, second displayed formula: after the second =, replace f by $\Gamma(f^*)$.
4. In the statement of Theorem 3.4, $\varepsilon\mu$ should be replaced by $\mu(\varepsilon^{-1/2}\,\cdot)$. The speed $\varepsilon^{-\beta}$ resulting from the application of the theorem on p. 1088 is correct, though.
5. First line of p. 1089: Replace $v_0^{1+\beta}$ by $v_0\varepsilon^{1+\beta}$. To make the estimate work for $t = 0$, confine ε to the finite interval [0,1] instead of $\mathbb{R}^+$ in line −4 of p. 1088.

References

[1] Jacquier, A., Pakkanen, M. S. and Stone, H. (2018). Pathwise large deviations for the rough Bergomi model. *J. Appl. Prob.* **55**, 1078–1092.

[2] Lifshits, M. (2012). *Lectures on Gaussian Processes*. Springer, Heidelberg.

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