BRIEF PROJECT REPORT ON THE START PROJECT "GEOMETRY OF STOCHASTIC DIFFERENTIAL EQUATIONS" (Y328)

1. Information on the research work

In the last ten years my research interests have been equally attracted by applied and pure questions in mathematics. It is worth mentioning that in applications the tractability of models always plays a major role, in particular in mathematical finance where real time evaluation is important. Therefore we have always tried – within this START project – to follow the four-step-procedure of understanding (in applied mathematics): having an idea – working it out for yourself – working it out for others – implementing it.

I still remember very well writing the research proposal for this START project in October 2005 in a Viennese restaurant: at this time I have been very fascinated (and I still am) by cubature methods, their rich geometric structures, and the chance to apply those methods to numerical problems of finite or infinite dimensional SDEs, which appear, for instance, in mathematical finance. On the other hand the geometric structure of diffusions, the complex interplay between two or more vector fields, have always been an ongoing source of interest and inspiration for me. Both questions, motivated and stimulated now even more by applications in finance, are still ongoing topics of my research. Today I would probably put more emphasis in the proposal on the problems from applications, which I have and had in mind, in order to justify the use of certain methods. One can say that I have learned this (important) message during and through this research project and I see that those research pieces went particularly well where problem, methods and implementation have played together in a smooth way.

The START proposal assigns three research fields to high order weak numerical schemes for S(P)DEs and cubature related algorithms (1, 3 & 4). Two fields of research are assigned to hypo-ellipticity and its counterpart, existence of finite dimensional realizations (2 & 5). Field 6 & 7 are assigned to one famous offspring of the theory of finite dimensional realizations: (affine) factor models, their calibration and their hedging properties.

Research fields 2 & 5 devoted to the structure theory of SPDEs led after three years of work to four articles in cooperation with Damir Filipović and Stefan Tappe from Vienna Institute of Finance (now Damir Filipović is at EPFL and Stefan Tappe at ETHZ) completely clarifying the geometric structure of jump diffusions on Hilbert spaces. The detour towards rough paths ("method of the moving frame") led to further unexpected results in existence and uniqueness theory of Banach space valued SDEs. Hypo-ellipticity results in the realm of this research have been obtained with my PhD-student Barbara Forster and Eva Lütkebohmert (then university of Bonn, now university of Freiburg).

Fields 1, 3 & 4 actually led to less research than expected, or – to be more precise – is leading to the expected results only now: Christian Bayer's PhD thesis, which was completed within the START project presents an SPDE version of cubature formulas. These ideas lead now to several projects where cubature methods, or related ones (like Ninomiya-Victoir methods) are applied to concrete SPDEs from mathematical finance. With Dejan Velusĉek and Kojiro Oshima we could establish a robust (meaning that the methods work indepedent of dimension, hence are also valid for SPDEs) generalization of Fujiwara's extrapolation scheme. The "method of the moving frame" provides another way towards establishing cubature methods, or treating hypo-ellipticity, for SPDEs – here several consequences are still unexplored. Somehow related, but with different methods from Malliavin calculus, are the numerical methods developed in the PhD thesis of Maria Siopacha (who was actually only shortly supported by the START project, but mainly by a graduate school in Vienna).

As a surprise, maybe to be expected, it turned out that the more concrete theory of factor models led to rich, interesting, important and applicable results, in particular in the field of affine processes and their relation to mathematical finance. The results on matrix-valued affine processes will be found in Christa Cuchiero's (foreseeably) excellent PhD thesis (fully written within the START project). Martin Keller-Ressel's extraordinary research work (as a PhD-student and as a Post-doc within the START project) focuses on affine processes and their applications in finance, e.g. interest rate theory together with Thomas Steiner in his PhD thesis, LIBOR market models together with the START-Post-Doc Antonis Papapantoleon, several questions on stochastic volatility, moment explosions, etc. Martin Keller-Ressel also contributed substantially to the solution of the regularity problem for affine processes.

Coming from Freiburg, where he wrote an excellent PhD-thesis, Antonis Papapantoleon worked as Post-Doc within the START project and brought his highly valuable expertise in the theory of exponential Lévy models into the project, leading, for instance, to one important joint publication on new approaches for LIBOR models.

Several new areas have been included within the START project, sometimes with immediate success, sometimes with possible future outcomes: Walter Schachermayer and myself got interested in optimal transport stimulated by joint reading of Cedric Villani's book on optimal transport. We could solve two of the problems posed in the book, one of them an important existence question. With view to this new field of research Gabriel Maresch (with a background in measure theory and set theory) was hired for the START project leading to further interesting research results in the realm of optimal transport. As START-Post-doc Evelina Shamarova was beginning to work on stochastic representations of Navier-Stokes or Burgers type equations, a work then successfully continued with Ana-Bela Cruzeiro in Lisbon.

Surprising, stimulating and still ongoing is the work in risk management, which was triggered by the SPDE results on the one hand and the offer of a consulting project within a large Austrian bank on the other hand. Within this project we could come up with theory, numerics and implementation of a scenario generator for the purposes of BASEL II internal modelling. Here it is also worth mentioning that there is ongoing cooperation with Tsuchiya Takahiro (Ritsumeikan university Kyoto), who worked as a START-Post-doc for some time.

Another unexpected detour was made towards simulated annealing and a new algorithm based on non-elliptic (but hypo-elliptic) basic geometric structures could be developed.

Finally three algorithms (cubature for HJM equations, polynomials processes, affine LIBOR models) developed within the START project have been included into PREMIA software at INRIA Paris.

2. Most important Results and (detailed) description of the research

The single most important result from a mathematical point of view is certainly the regularity of affine processes on canonical and general state spaces. It was obtained through results of Martin Keller-Ressel's PhD thesis and an unusual application of the "method of the moving frame", see [28] and the follow-up paper [29].

The single most impacting result might be the characterization of positive semidefinite matrix valued processes in [8], since it continues towards the direction of jump processes outstanding work in stochastic analysis by Marie-France Bru on Wishart processes and deep, far-reaching analytic work on symmetric cones by Jacques Faraut and Adam Koranyi. Several mathematical follow-up works on symmetric cones in finite or infinite dimension are in preparation, and, most important, several works on applications in mathematical finance are considered.

[39] is a good example of mathematics in the sense that it answers a question which has not been asked by oneself but by (many) others. There a clear answer is given how c-monotone transport plans are related to solutions of optimal transport problems.

A new approach to LIBOR modelling, which guarantees within a wide class of models tractability (with respect to all forward measures) is described in all detail in [27].

Finally it has been proved in [3] that one can use cubature methods, i.e. high order weak approximation schemes, in the realm of SPDEs. In [16] fundamental results on existence, uniqueness, stability and numerics for SPDEs with jump-diffusion characteristics have been proved.

All obtained results can be categorized in several different groups, some of them related to research fields from the START proposal: structure theory of SPDEs (related to field 2, 5, 6, 7), factor models in mathematical finance (related to field 6, 7), high order weak approximation schemes for S(P)DEs (related to field 1, 3, 4). Overviewing the proposed fields of research in the research proposal "Geometry of stochastic differential equations" and the results of three years of intensive and concentrated work leads to the following more detailed picture:

2.1. **Structure theory of SPDEs.**

Questions like existence, uniqueness, positivity, finite dimensional realizations of SPDEs driven by general jump processes (Lévy processes, or, more generally, Poisson random measures) have been answered in full generality, even with an unexpected detour towards rough paths, in a series of papers, see [16, 17, 18, 19, 48]:

In [16] existence, uniqueness, stability and a bit of numerics for general (functionally dependent) jump diffusion SPDEs is shown by means of the method of the moving frame. The results allow for a treatment of term structure equations, for instance in interest rate theory, which was done in [17] together with a complete classification of positive term structure models. The question of finite dimensional realizations is addressed and solved in [18].

In [19] hypo-ellipticity in a finite (or infinite) dimensional setting is treated in the presence of jumps including the appropriate Greeks' formulas.

Related to finite dimensional realizations but with a different focus is the article [5], where a most general equivalence between deterministic and stochastic invariance is shown by quite simple methods.

2.2. **Factor models in mathematical finance.** Questions on factor models have been considered mainly, but not exclusively, in the realm of affine processes: here several layers of results could be obtained. First it has been proved that the fundamental assumption of regularity is in fact redundant making the theory of affine processes (and their applications) more transparent, see [28]. Second affine processes have been characterized on important state spaces, such as positive semidefinite matrices or symmetric cones, see [8, 9]. Third the class of affine processes has been extended towards the tractable class of polynomial processes, see [11]. Fourth, contributions in areas like duality theory for pricing, stochastic volatility, interest rates and applications of affine processes, have been worked out, see [12, 13, 14, 15, 20, 21, 23, 24, 26, 27, 30, 32, 36]:

[8] provides the mathematical foundation for stochastically continuous affine processes on the cone of positive semidefinite symmetric matrices (i.e. covariance matrices). This analysis has been motivated by multivariate stochastic volatility models and fixed-income models with stochastically correlated risk factors. The authors completely characterize this class of processes and establish a relation to infinitely decomposable Markov processes.

The survey article [10] presents theoretical and empirical aspects of affine term structure models. The authors explain their relations to affine processes and show how the analytic tractability of affine models in turn can be exploited for pricing and model calibration.

In [11] the class of so-called "polynomial" Markov processes is introduced. These processes have the nice property that moments can be computed via matrix exponentials. This gives rise to new techniques for option pricing, in particular for variance reduction techniques in Monte Carlo simulations. Such a pricing algorithm for a stochastic volatility model with jumps has been implemented in the software package PREMIA.

[12] discusses conditions under which Fourier transform valuation formulas for path-dependent options with arbitrary payoff functions are valid. As an application, lookback options in Lévy- and stochastic volatility models are priced.

By means of the Wiener-Hopf factorization, the authors of [13] provide formulas for the valuation of path-dependent options (e.g. one-touch and lookback options) in Lévy models.

[15] describes the mathematical framework for the study of the duality principle in option pricing. The authors consider models where prices evolve as general exponential semimartingales and provide a characterization of the dual process under the dual measure. In [14], the study of the duality principle is extended to options that depend on several assets. Here, the dual measures are constructed via an Esscher transformation.

[30] discusses the shape of yield curves in interest rate models, where the short rate is given by a one-dimensional, time-homogeneous affine factor process. The authors show that the yield curve can take only three possible shapes: increasing, decreasing, or with a single maximum.

[25] contains results on the convexity of solutions of certain differential equations, generalizing known results from ODE theory. The results were motivated by differential equations of generalized Riccati type, which appear in relation with affine processes.

The article [24] treats the valuation of forward-start options in the Barndorff-Nielsen-Shephard stochastic volatility model. The authors derive the conditional characteristic function of the log-price and apply Fourier techniques to calculate option prices.

The short article [20] summarizes results on moment explosions in stochastic volatility models, and explains the relation of moment explosions to the shape of the implied volatility surface.

Several publications of Martin Keller-Ressel are devoted to the theory of affine processes and their applications to stochastic volatility modeling: In his PhD thesis [22] he presents sufficient conditions for regularity (i.e. differentiability of the characteristic function) and gives a new proof of the characterization result of Duffie, Filipović and Schachermayer (2003). The regularity result was subsequently extended in [28] where the authors show in full generality that every stochastically continuous affine process is regular. This surprising result shows that the regularity assumption that was previously made in the literature is automatically fulfilled. In [23] the properties of stochastic volatility models based on affine processes are further explored: The author derives results on moment explosions and the long-term behavior of such stochastic models, and discusses the connection to the implied volatility surface produced by the model.

In [27] affine processes are applied to LIBOR modeling. The authors construct a LIBOR model that guarantees positive LIBOR rates and stays tractable under all forward measures. The affine LIBOR model proposed in [27] is currently implemented by Martin Keller-Ressel as a part of the PREMIA software platform developed by INRIA in Paris, see `http://www-rocq.inria.fr/mathfi/Premia/index.html`.

In [36] Antonis Papapantoleon compares this new model to existing approaches, such as classical LIBOR market models and forward price models.

[31] is a short note on the representation of implied volatility as the time-average of weighted expectations of local or stochastic volatility.

In [26] the authors consider the exact pricing of derivatives written on the discrete realized variance of an underlying security. They show that the usual approach of approximating realized variance by its continuous-time limit, the quadratic variation, may lead to considerably different prices.

In [32] explicit valuation formulas for a path-dependent interest rate derivative, namely an option on the composition of LIBOR rates, in an HJM-type and in a LIBOR-type model are derived. The formulas are based on Fourier methods for option pricing.

2.3. **High order weak approximation schemes.** Questions on high-order weak approximation schemes could be solved for S(P)DEs (convergence proof for SPDEs, general extrapolation schemes à la Fujiwara) and a new algorithm for the calculation of Greeks could be presented, see [3, 16, 35, 47, 48]:

In [3] a setting of vector fields (and jump characteristics) could be described such that cubature formulas converge (even with high order rates of weak convergence). It was possible to find a setting which works without restrictive assumptions on the involved semigroup.

In [16] the same point of view is taken up but by means of the method of the moving frame, which makes the approach analytically easier.

In [35] a general Fujiwara-type extrapolation scheme with high order weak convergence rates is presented with all details on the algebraic structures behind. Such schemes also work for SPDEs.

In [47] Malliavin calculus methods are used to describe and express coefficients of Taylor-like expansion of option prices in mathematical finance, a method related to the so-called Watanabe expansion.

In [48] another approach to stochastic or rough partial differential equations is presented based on the method of the moving frame. In some sense it is the most general version of obtaining Banach space valued solutions of stochastic differential equations.

2.4. **Optimal transport.** In [39] it is shown that c-cyclically monotone transport plans optimize the Monge-Kantorovich transportation problem under an additional measurability condition. This measurability condition is always satisfied for finitely valued, lower semi-continuous cost functions. In particular, this yields a positive answer to Problem 2.25 in Cedric Villani's book.

In [38] an easy counter-example to Problem 7.20 from Cedric Villani's book on mass transport is given.

In [4] the Monge-Kantorovich transport problem is considered in a purely measure theoretic setting, i.e. without imposing continuity assumptions on the cost function. It is known that transport plans which are concentrated on c-monotone sets are optimal, provided the cost function c is either lower semi-continuous and finite, or continuous and may possibly attain the value infinity. This is even true in a more general setting, in particular for merely Borel measurable cost functions provided that the set $\{c = \infty\}$ is the union of a closed set and a negligible set.

2.5. **Simulated Annealing.** A new algorithm for simulated annealing in subriemannian geometric structures is presented in [2]: Ornstein-Uhlenbeck processes associated to hypo-elliptic diffusion processes on finite-dimensional Lie groups are constructed and Poincaré type inequalities are proved. These results are then applied to obtain a hypo-elliptic equivalent of standard results on cooling schedules for simulated annealing on compact homogeneous spaces, see [2].

2.6. **Risk management.** A new dynamic approach to scenario generation for the purposes of risk management in the banking industry has been provided. Ideas from conventional techniques – like historical and Monte Carlo simulation – are connected to come up with a hybrid method that shares the advantages of standard procedures but eliminates several of their drawbacks. Instead of considering the static problem of constructing one or ten-day-ahead distributions for vectors of risk factors, the problem is embedded into a dynamic framework, where any time horizon can be consistently simulated. Additionally, standard models from mathematical finance for each risk factor are used, whence bridging the worlds of trading and risk management. The approach is based on stochastic (partial) differential equations, like the HJM-equation or the Black-Scholes equation, governing the time evolution of risk factors, on an empirical calibration method to the market for the chosen SDEs, and on an Euler scheme (or high order schemes) for the numerical evaluation of the respective SDEs. The empirical calibration procedure presented in the paper can be seen as the SDE-counterpart of the so-called Filtered Historical Simulation method; the behavior of volatility stems in our case out of the assumptions on the underlying SDEs. All results see [34].

2.7. **Premia.** Algorithms presented in [3, 11, 27] have been successfully implemented into a large scale software package managed and produced at INRIA (Paris).

In [3] it is proved that one can use cubature methods, i.e. high order weak approximation schemes, in the realm of SPDEs. The resulting algorithms have been implemented for the HJM equation.

In [11] the class of so-called "polynomial" Markov processes is introduced. These processes have the nice property that moments can be computed via matrix exponentials, which gives rise to new techniques for option pricing, in particular for variance reduction techniques in Monte Carlo simulations. Such a pricing algorithm for a stochastic volatility model with jumps has been implemented in the software package PREMIA.

In [27] affine processes are applied to LIBOR modeling. The authors construct a LIBOR model that guarantees positive LIBOR rates and stays tractable under all forward measures. The affine LIBOR model proposed in [27] is currently implemented by Martin Keller-Ressel as a part of the PREMIA software platform.

2.8. **BMV-conjecture.** The main task of this work has been to provide estimations and a strategy to calculate the density of a certain measure on the positive real line within a given error bound. Implementations and further information can be found on `http://www.math.ethz.ch/~ggeorg/master/index.html`.

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