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Geometry of Interest Rates: the Research Unit of Financial and Actuarial Mathematics at the Vienna University of Technology

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by Josef Teichmann (/author-index/l/lintner-will9)

When you think of pushing a wheel of fortune you probably imagine a fancy prize like a bag full of money or a cruise through the Caribbean Sea. But would you ever think of

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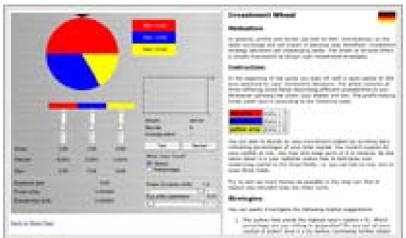
Financial Mathematics? Most of us would not. But the job of a Financial and Actuarial Mathematician offers a wide range of possibilities. In addition to the rather classical part of this discipline, the calculation of life insurance, exciting fields of activity have been arising: risk management for markets in banking and credit, consulting in the field of capital investments, actuarial calculations for pension funds, definition of insurance charges, asset-liability-management, or developing investment strategies for the stock exchange regarding interest rates.


"Wheel of fortune" for investments.

(http://www.fam.tuwien.ac.at/public/simulation/wheel_en.html) Financial Mathematics is a flourishing area of modern science, applying profound knowledge of pure mathematics to problems of financial economics. In general, profits and losses (as well as their distributions) on the stock exchange are not known in advance and, therefore, investment strategy decisions are challenging tasks. The "wheel of fortune (http://www.fam.tuwien.ac.at/public/simulation/wheel_en.html)" of the Research Unit of Financial and Actuarial Mathematics at the Vienna University of Technology (TU Vienna) offers a simple framework to design such investment strategies.

The work of the research group at TU Vienna is one of the outstanding examples of how fundamental research and practical applications can be successfully combined. Josef Teichmann, a young scientist within this research group who was recently awarded the Austrian START-Prize, explains in the following article the work and the ambitions of the institute.

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Geometry of interest rates

The START Prize is awarded by the Austrian Science Fund on behalf of the Federal Ministry for Education, Science and Culture (bm:bwk). It enables young researchers to take a long-term perspective with extensive financial support (up to €1.2 million for a period of six years) to plan their research and build up their own research group.

My project, which was awarded one of the five START prizes in 2006, is called "Geometry of Stochastic Differential Equations." Stochastic differential equations are

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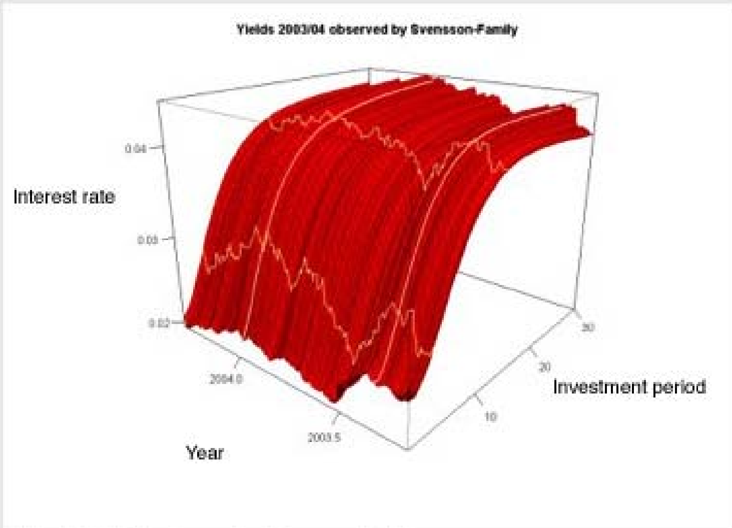
a key tool in modeling uncertain time evolutions in technology, science, or economics. As often in science, a project is best explained by giving examples of problems: in the present case, Interest Rates.

Josef Teichmann

Interest rates quantify the time value of money. More precisely, every day at major stock exchanges it is decided through trading how much one has to pay today to obtain one unit of currency three months, one year, five years, or thirty years later. This can be expressed by a compound rate - the interest rate for the maturity in question - saying that one will receive, e.g., an annual interest rate of 2.5 percent if one invests for three years. Looking at the daily rates for different investment periods and graphing these rates in a coordinate system, as in the following picture, defines a surface. In Interest Rate theory, as in other areas of applied mathematics, one now has to deal with the unusual geometric task of describing this rough surface. The following picture shows the interest rate evolution between 2003 and 2004. The rough curves (the yellow lines that cross the curved surface) are two examples of the daily changes in interest rates, while the smooth (red) curves represent the interest rates for different investment periods - the rates increasing for longer investment periods.

The notion "Svensson-family" refers to a well-specified six-parameter family of curves, which are applied to map the market data to the curves shown in the picture.



Yields 2003/2004 observed by Svensson-Family.

Coming from a background in geometry, I learned mathematics as an abstract science where mere curiosity, the wish to understand, and a feeling for aesthetics govern the development of theory. Surprisingly, this nearly artistic background laid the basis for dealing with modern interest rate theory and enabled us in recent years to comprehend these rough surfaces better.

The analysis of such surfaces in a qualitative and quantitative way is one of the cornerstones of my START project. The theory quickly leads to stochastic differential equations with values in Hilbert spaces. The practical implementations lead to parametrized families of curves in order to fit the daily rates in banks or investment houses. Frobenius Theorem for unbounded vector fields and analysis of involutive bundles on Hilbert spaces are the links between the mathematical framework of stochastic equations and the practical need for finitely parametrized families of curves.

However, it was not by chance that I came in contact with economics as a so-called pure mathematician: it was due to the growing visibility of the research group led by Walter Schachermayer at TU Vienna that I became interested in financial mathematics. Since 1998 Walter Schachermayer has been a full professor of financial

Walter Schachermayer © TU Wien

and actuarial mathematics at TU Vienna, representing the subject of financial mathematics with his background in stochastic processes and financial mathematics. Due to the Wittgenstein prize awarded to him in 1998 by the Austrian Science Fund (FWF), he has built a highly visible research group in financial mathematics. This group is dealing successfully with questions like no arbitrage theory, portfolio optimization, and transaction costs in theoretical and practical directions.

Since the Nobel Prize-winning research of F. Black, R. Merton, and M. Scholes on the pricing of option contracts, the field of financial mathematics has developed into a very active area of applied mathematics. In 1973 Black, Merton, and Scholes described a way to uniquely determine the price of a European call contract. Such a contract written on an underlying stock represents the right, but not the obligation, to buy one share of the stock - at a pre-specified time in the future - for a price that is fixed today. European call contracts are traded on all major stock exchanges and their prices are determined by supply and demand. More surprising, though, is coming up with a method to relate the price of the stock to the price of the option, as Black, Merton, and Scholes did.

Their trailblazing insight was based on careful modeling of stock markets; in particular, they rigorously incorporated trading into their model. Having a clear idea how to deal with traded portfolios, it is easy to think about portfolios which bring - sometime in the future - risk-free gains (i.e., no losses whatsoever can appear). Such portfolios are called arbitrage portfolios and to exclude them is the paradigm of modern mathematical finance: No Arbitrage. This simple principle, together with some special assumptions on their model, allowed Black, Merton, and Scholes to conclude their formula in 1973.

Seen through the eyes of a mathematician, not only the insight (how to set up the model), but the right tools (how to work out the model) were available at the right time. Somehow, they were available at just at the right time: the highly challenging theory of stochastic processes had been developed in the '50s and '60s guided by physical motivations but without any economic applications in mind. Yet this theory fit the needs of financial modeling perfectly. Pioneering work was done in the '40s in Japan by K. Itô, who, now at the age of 94 years, was recently awarded the 2006 Gauss prize for his achievements by the International Mathematical Union. It is the pride of mathematical finance that the earliest work in the direction of stochastic processes (which unfortunately remained without much impact for a long time) was done by L. Bachelier in 1900 in Paris on the problem of pricing options. The considerations leading to his pricing formula are a masterpiece of early financial mathematics, and are still valid. In particular, his work anticipated the 1905 theory of A. Einstein and M. Smoluchovski on Brownian motion.

The Black-Merton-Scholes formula constitutes the basic formula of financial mathematics and is widely applied every day in the financial industry. Nevertheless, the validity of this formula is rather limited and several extensions of the Black-Merton-Scholes setting have been worked out in the last 30 years. This does not mean that the paradigm of no arbitrage has changed; rather, assumptions are weakened or adapted to a specific situation. For instance, this may concern the introduction of transaction costs or risks to default. Many of the extensions share one or both of the following drawbacks:

- the prices of options or other contracts are not unique anymore, and

- the calculation of prices, if possible to determine, requires very subtle methods of scientific computing.

It is worth mentioning that one reason for success of the Black-Merton-Scholes formula is that every pocket calculator can deal with it.

In Walter Schachermayer's group at TU Vienna, several of the questions raised were dealt with successfully: for instance, how to obtain unique prices by utility considerations, or how to introduce transaction costs, or how to deal with interest rates. Through the geometric nature of the problem, I came in contact with his group in February 2000. In July 2000, I started my postdoctoral study there, a very fruitful period due to the good scientific atmosphere and the well-posed questions. Since then, several other research interests have arisen, such as numerical computations in high-dimensional systems for the purposes of mathematical finance or optimal transportation. Here again, geometric reasoning seems to be the key to better comprehension of algorithms.


Uwe Schmock © TU Wien

In 2003 Uwe Schmock from ETH Zurich was appointed full professor in a newly founded professorship at the Department of Financial and Actuarial Mathematics at TU Vienna. As a driving force, he set up a cooperation between the Department and Austria's largest bank, Bank Austria-Creditanstalt (BA-CA). This cooperation is realized through the Christian Doppler Research Association, an institution which makes additional governmental funds available for the research. The research carried out at the Christian Doppler Laboratory for Portfolio Risk Management (PRisMa Lab) combines academic, methodological research with a strong input from and interaction with its founding industry partner, BA-CA, for their mutual benefit. The laboratory concentrates on integrated financial risk management, taking dependence structures - in particular portfolio effects - into account. Recently, the PRisMa Lab was extended by a module cooperation with the Austrian Federal Financing Agency for quantification of counterparty risk for exotic swaps.

Today the Department of Financial and Actuarial Mathematics is involved in a wide range of research topics from academic considerations to practical implementations.

Further information can be found at the following Web sites:

Walter Schachermayer: http://www.fam.tuwien.ac.at/~wschach (http://www.fam.tuwien.ac.at/%7Ewschach)
Josef Teichmann: http://www.fam.tuwien.ac.at/~jteichma (http://www.fam.tuwien.ac.at/%7Ejteichma)
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