23. Juni 2008
105.057 Finanzmathematik 2: zeitstetige Modelle, Schachermayer

Dauer 90 Minuten, alle Unterlagen sind erlaubt

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1. Consider a derivative with payoff (5 Pkt.)

$$X = \begin{cases} 0 & \text{if } S_T < A, \\ S_T - A & \text{if } A \le S_T \le B, \\ C - S_T & \text{if } B \le S_T \le C, \\ 0 & \text{if } S_T > C. \end{cases}$$

   Here S is the price of the underlying, and A, C are real numbers with $0 < A < C$ and $B = \frac{1}{2}(A + C)$.

   (a) Draw a payoff diagram and show that this contingent claim can be replicated by a static portfolio of European call options.

   (b) Suppose that we want to replicate X in the Black-Scholes model by investing only in the bank account and the stock. How many units of stock are in a replicating portfolio at time $0 \le t \le T$? (Express the answer in terms of the distribution function of the standard normal distribution.)

2. Determine the price of the "cash or nothing" option with strike K and payoff (5 Pkt.)

$$\mathbf{1}_{\{S_T \ge K\}}$$

   in the Black-Scholes model.

3. Let $c(t, S_t)$ and $p(t, S_t)$ denote the prices of a European call and a put option with maturity T, underlying S (non-dividend-paying), and strike K, respectively. Prove the following assertions, assuming a constant interest rate r. Do *not* use a portfolio argument or the Black-Scholes formula, but the risk-neutral pricing formula. (5 Pkt.)

   (a) (Put-call parity)
$$c(0, S_0) - p(0, S_0) = S_0 - \mathrm{e}^{-rT} K,$$

   (b)
$$(S_0 - \mathrm{e}^{-rT} K)_+ \le c(0, S_0) \le S_0.$$

   (Hint: You may use (a) to prove the lower estimate in (b).)