1. Consider a derivative with payoff

\[ X = \begin{cases} 
0 & \text{if } S_T < A, \\
S_T - A & \text{if } A \leq S_T \leq B, \\
C - S_T & \text{if } B \leq S_T \leq C, \\
0 & \text{if } S_T > C.
\end{cases} \]

Here \( S \) is the price of the underlying, and \( A, C \) are real numbers with \( 0 < A < C \) and \( B = \frac{1}{2}(A + C) \).

(a) Draw a payoff diagram and show that this contingent claim can be replicated by a static portfolio of European call options.

(b) Suppose that we want to replicate \( X \) in the Black-Scholes model by investing only in the bank account and the stock. How many units of stock are in a replicating portfolio at time \( 0 \leq t \leq T \)? (Express the answer in terms of the distribution function of the standard normal distribution.)

2. Determine the price of the “cash or nothing” option with strike \( K \) and payoff

\[ 1_{\{S_T \geq K\}} \]

in the Black-Scholes model.

3. Let \( c(t, S_t) \) and \( p(t, S_t) \) denote the prices of a European call and a put option with maturity \( T \), underlying \( S \) (non-dividend-paying), and strike \( K \), respectively. Prove the following assertions, assuming a constant interest rate \( r \). Do not use a portfolio argument or the Black-Scholes formula, but the risk-neutral pricing formula.

(a) (Put-call parity)

\[ c(0, S_0) - p(0, S_0) = S_0 - e^{-rT}K, \]

(b)

\[ (S_0 - e^{-rT}K)_+ \leq c(0, S_0) \leq S_0. \]

(Hint: You may use (a) to prove the lower estimate in (b).)