

Name:

Mat.Nr.:

Bitte keinen Rotstift verwenden!

**Lebensversicherungsmathematik**  
**(Vorlesungsprüfung)**  
**4. Mai 2018**  
**Univ.Prof. Rheinländer**

Dauer: 90 Minuten

Unterlagen: ein beidseitig handbeschriebener A4-Zettel sowie ein nichtprogrammierer Taschenrechner sind erlaubt

Anmeldung zur mündlichen Prüfung im FAM-office,  
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Bsp.	Max.	Punkte
1	8	
2	7	
3	8	
$\Sigma$	23	

Schriftlich:

Assistent:  
Dragana Radojčić

Mündlich:

**Gesamtnote:**

1. Rechnen Sie dieses Beispiel ohne Sterbe- oder Leibrententafeln.

(8 Pkt.)

(a) Zeigen Sie

$${}_n p_x d\ddot{a}_{\overline{n}|} + \sum_{k=0}^{n-1} (1 - v^{k+1}) {}_k p_x q_{x+k} = 1 - A_{x:\overline{n}|}, \quad x, n \in \mathbb{N}.$$

(b) Es sei  $T_0$  exponentialverteilt mit Parameter  $\lambda > 0$ , d.h. die Verteilungsfunktion von  $T_0$  ist gegeben durch

$$F_0(y) = (1 - \exp(-\lambda y)) \mathbb{1}_{[0, \infty)}(y), \quad y \in \mathbb{R}.$$

Bestimmen Sie die Verteilungsfunktion  $F_x$  von  $T_x$  und  $\mathbb{E}[T_x]$  für alle  $x \geq 0$ . Überprüfen Sie weiters ob Annahme A (Annahme A1 und Annahme A2) erfüllt ist. Ist Annahme B erfüllt?

(c) Gegeben sind zwei Zufallsvariablen  $T_x$  und  $T_y$ , die die Restlebenszeit von zwei Personen im Alter  $x$  bzw. Alter  $y$  modellieren. Die gemeinsame Dichte von  $T_x$  und  $T_y$  sei gegeben durch

$$f(s, t) = C \cdot (50^2 - (s - t)^2), \quad \text{falls } s \in [0, 50] \text{ und } t \in [0, 50],$$

und 0 sonst (Achtung:  $T_x$  und  $T_y$  sind nicht unabhängig). Hier ist  $C > 0$  eine Normierungskonstante. Bestimmen Sie  $C$ .

(d) Sei  $T_x$  exponentialverteilt mit Parameter  $\lambda > 0$ , d.h. die Verteilungsfunktion von  $T_x$  ist gegeben durch  $F(y) = 1 - \exp(-\lambda y)$ ,  $y \geq 0$ . Weiters sei eine positive Zinsintensität  $\delta$  gegeben. Zeigen Sie

$$\overline{A}_x = \frac{\lambda}{\lambda + \delta}.$$

Sei  $T_x$  die Restlebenszeit einer  $x$ -jährigen Person,  $K_x = \lfloor T_x \rfloor$  und  $S_x = T_x - K_x$ . Oft werden Annahmen über die Verteilung von  $K_x$  und  $S_x$  getroffen:

- Annahme A1: für  $x \in \mathbb{N}$  ist  $S_x$  unabhängig von  $K_x$  und  $S_x$  ist gleichverteilt im Intervall  $[0, 1)$ .
- Annahme A2: für  $x \in \mathbb{N}$  und  $s \in [0, 1)$  ist  ${}_s q_x = s q_x$ .
- Annahme B: für  $x \in \mathbb{N}$  und  $s \in [0, 1)$  ist  $s \mapsto \mu_{x+s}$  konstant.

2. Zeigen Sie folgende Näherung für die Varianz des Nelson-Aalen Schätzers

(7 Pkt.)

$$\text{Var}(\tilde{S}(t)) \approx \tilde{S}(t)^2 \sum_{t_i \leq t} \frac{d_i(n_i - d_i)}{n_i^3}.$$

Wie kann man daraus Konfidenzintervalle für  $\tilde{S}(t)$  bekommen?

3. Verwenden Sie für dieses Beispiel die Werte aus Tabelle 1. Gehen Sie von  $r = 1\%$  und einem Höchstalter von 52 aus. (8 Pkt.)

$x$	$l_x$
49	1700
50	1500
51	1100
52	700

Tabelle 1: Toy-Sterbetafel

- (a) Verwenden Sie Kommutationszahlen, um  $\ddot{a}_{49:\overline{2}|}$  zu berechnen.
- (b) Sei  $x = 50$ . Verwenden Sie Annahme A (Annahme A1 und Annahme A2) um die Verteilungsfunktion  $t \mapsto F_x(t)$  von  $T_x$  zu bestimmen (für alle  $t \in \mathbb{R}$ !).
- (c) Sei  $x = 50,5$ . Verwenden Sie Annahme B, um die Verteilungsfunktion  $t \mapsto F_x(t)$  von  $T_x$  zu bestimmen (für alle  $t \in \mathbb{R}$ !).
- (d) Eine 49-jährige Person A und eine 50-jährige Person B kaufen eine ewige Ablebensversicherung, die am Ende des Todesjahres von B 700 € ausgezahlt, falls A dann noch am Leben ist. Ist A am Ende des Todesjahres von B nicht mehr am Leben, wird nichts ausbezahlt. Unter der Annahme, dass die Restlebenszeiten beider Personen unabhängig sind, berechnen Sie die NEP dieser Versicherung.

*Hinweis zu (b) und (c):* Machen Sie Fallunterscheidungen!

**Sterbetafel 2010/2012 männlich und weiblich zusammen**

Genaueres Alter (am x-ten Geburts- tag) in Jahren	Sterbe- wahrscheinlichkeit im Altersintervall x bis x+1	Überlebende im Alter x	Gestorbene im Altersintervall x bis x+1	Von den Überlebenden im Alter x		Fernere Lebens- erwartung im Alter x in Jahren	Genaueres Alter (am x-ten Geburts- tag) in Jahren	Sterbe- wahrscheinlichkeit im Altersintervall x bis x+1	Überlebende im Alter x	Gestorbene im Altersintervall x bis x+1	Von den Überlebenden im Alter x		Fernere Lebens- erwartung im Alter x in Jahren
				bis x+1	insgesamt						bis x+1	insgesamt	
				noch zu durch- lebende Jahre							noch zu durch- lebende Jahre		
x	q(x)	l(x)	d(x)	L(x)	T(x)	e(x)	x	q(x)	l(x)	d(x)	L(x)	T(x)	e(x)
0	0,0035623	100000,00	356,23	99.684	8.067.528	80,68	50	0,0028230	96628,47	272,78	96.492	3.125.168	32,34
1	0,0002311	99643,77	23,03	99.632	7.967.844	79,96	51	0,0031253	96355,69	301,14	96.205	3.028.676	31,43
2	0,0001844	99620,74	18,37	99.612	7.868.212	78,98	52	0,0034551	96054,55	331,88	95.889	2.932.470	30,53
3	0,0001429	99602,37	14,23	99.595	7.768.600	78,00	53	0,0038143	95722,66	365,11	95.540	2.836.582	29,63
4	0,0001096	99588,14	10,91	99.583	7.669.005	77,01	54	0,0042084	95357,55	401,30	95.157	2.741.042	28,74
5	0,0000858	99577,22	8,55	99.573	7.569.422	76,02	55	0,0046432	94956,25	440,90	94.736	2.645.885	27,86
6	0,0000728	99568,68	7,25	99.565	7.469.850	75,02	56	0,0051288	94515,35	484,75	94.273	2.551.149	26,99
7	0,0000689	99561,43	6,86	99.558	7.370.284	74,03	57	0,0056639	94030,59	532,58	93.764	2.456.876	26,13
8	0,0000695	99554,57	6,92	99.551	7.270.726	73,03	58	0,0062461	93498,01	584,00	93.206	2.363.112	25,27
9	0,0000707	99547,64	7,04	99.544	7.171.175	72,04	59	0,0068749	92914,01	638,77	92.595	2.269.906	24,43
10	0,0000732	99540,60	7,28	99.537	7.071.631	71,04	60	0,0075429	92275,24	696,02	91.927	2.177.311	23,60
11	0,0000808	99533,32	8,05	99.529	6.972.094	70,05	61	0,0082426	91579,21	754,85	91.202	2.085.384	22,77
12	0,0000955	99525,27	9,51	99.521	6.872.565	69,05	62	0,0089571	90824,36	813,52	90.418	1.994.182	21,96
13	0,0001197	99515,76	11,91	99.510	6.773.044	68,06	63	0,0096956	90010,84	872,71	89.574	1.903.765	21,15
14	0,0001581	99503,85	15,73	99.496	6.673.535	67,07	64	0,0104493	89138,13	931,43	88.672	1.814.190	20,35
15	0,0002101	99488,12	20,90	99.478	6.574.039	66,08	65	0,0112182	88206,70	989,52	87.712	1.725.518	19,56
16	0,0002724	99467,22	27,10	99.454	6.474.561	65,09	66	0,0120090	87217,17	1047,39	86.693	1.637.806	18,78
17	0,0003411	99440,12	33,92	99.423	6.375.107	64,11	67	0,0128721	86169,78	1109,18	85.615	1.551.112	18,00
18	0,0004097	99406,20	40,73	99.386	6.275.684	63,13	68	0,0138399	85060,60	1177,23	84.472	1.465.497	17,23
19	0,0004689	99365,47	46,59	99.342	6.176.298	62,16	69	0,0149172	83883,37	1251,30	83.258	1.381.025	16,46
20	0,0005095	99318,88	50,60	99.294	6.076.956	61,19	70	0,0161495	82632,07	1334,47	81.965	1.297.767	15,71
21	0,0005283	99268,28	52,45	99.242	5.977.663	60,22	71	0,0175720	81297,60	1428,56	80.583	1.215.802	14,95
22	0,0005252	99215,84	52,11	99.190	5.878.421	59,25	72	0,0192313	79869,03	1535,99	79.101	1.135.219	14,21
23	0,0005088	99163,73	50,46	99.139	5.779.231	58,28	73	0,0211823	78333,05	1659,27	77.503	1.056.118	13,48
24	0,0004919	99113,27	48,76	99.089	5.680.092	57,31	74	0,0234471	76673,77	1797,78	75.775	978.615	12,76
25	0,0004807	99064,52	47,62	99.041	5.581.003	56,34	75	0,0261030	74875,99	1954,48	73.899	902.840	12,06
26	0,0004745	99016,89	46,98	98.993	5.481.963	55,36	76	0,0292000	72921,51	2129,31	71.857	828.941	11,37
27	0,0004708	98969,91	46,60	98.947	5.382.969	54,39	77	0,0328535	70792,20	2325,77	69.629	757.084	10,69
28	0,0004703	98923,31	46,53	98.900	5.284.023	53,42	78	0,0371406	68466,43	2542,88	67.195	687.455	10,04
29	0,0004752	98876,78	46,98	98.853	5.185.123	52,44	79	0,0420840	65923,55	2774,33	64.536	620.260	9,41
30	0,0004857	98829,80	48,00	98.806	5.086.269	51,46	80	0,0477560	63149,22	3015,76	61.641	555.724	8,80
31	0,0005026	98781,80	49,65	98.757	4.987.463	50,49	81	0,0542112	60133,47	3259,91	58.504	494.082	8,22
32	0,0005256	98732,16	51,89	98.706	4.888.707	49,51	82	0,0615230	56873,56	3499,03	55.124	435.579	7,66
33	0,0005550	98680,26	54,77	98.653	4.790.000	48,54	83	0,0696403	53374,53	3717,02	51.516	380.455	7,13
34	0,0005914	98625,49	58,33	98.596	4.691.347	47,57	84	0,0785868	49657,51	3902,42	47.706	328.939	6,62
35	0,0006313	98567,16	62,22	98.536	4.592.751	46,60	85	0,0885201	45755,08	4050,24	43.730	281.232	6,15
36	0,0006751	98504,94	66,50	98.472	4.494.215	45,62	86	0,0995412	41704,84	4151,35	39.629	237.502	5,69
37	0,0007244	98438,44	71,31	98.403	4.395.743	44,65	87	0,1120099	37553,49	4206,36	35.450	197.873	5,27
38	0,0007816	98367,13	76,89	98.329	4.297.341	43,69	88	0,1258038	33347,13	4195,19	31.250	162.423	4,87
39	0,0008491	98290,25	83,46	98.249	4.199.012	42,72	89	0,1409640	29151,93	4109,37	27.097	131.173	4,50
40	0,0009311	98206,78	91,44	98.161	4.100.763	41,76	90	0,1573921	25042,56	3941,50	23.072	104.076	4,16
41	0,0010317	98115,34	101,23	98.065	4.002.602	40,79	91	0,1750614	21101,06	3693,98	19254	81004	3,84
42	0,0011524	98014,11	112,95	97.958	3.904.538	39,84	92	0,1938233	17407,08	3373,90	15720	61750	3,55
43	0,0012920	97901,16	126,49	97.838	3.806.580	38,88	93	0,2137463	14033,18	2999,54	12533	46030	3,28
44	0,0014516	97774,68	141,93	97.704	3.708.742	37,93	94	0,2344209	11033,64	2586,52	9740	33497	3,04
45	0,0016304	97632,75	159,18	97.553	3.611.038	36,99	95	0,2560990	8447,12	2163,30	7365	23756	2,81
46	0,0018282	97473,56	178,20	97.384	3.513.485	36,05	96	0,2787092	6283,82	1751,36	5408	16391	2,61
47	0,0020451	97295,36	198,98	97.196	3.416.101	35,11	97	0,3018203	4532,46	1367,99	3848	10983	2,42
48	0,0022826	97096,39	221,63	96.986	3.318.905	34,18	98	0,3255082	3164,47	1030,06	2649	7134	2,25
49	0,0025423	96874,76	246,29	96.752	3.221.919	33,26	99	0,3493887	2134,41	745,74	1762	4485	2,10
							100	1,0000000	1388,67	1388,56	2723	2723	1,96

## Barwerte einer lebenslang vorschüssigen Rente vom Betrag 1 nach der Sterbetafel 2010/2012 männlich und weiblich zusammen

Genaueres Alter in Jahren	Zinsfuß															Genaueres Alter in Jahren										
	0,0%	0,5%	1,0%	1,5%	2,0%	2,5%	3,0%	3,5%	4,0%	4,5%	5,0%	5,5%	6,0%	6,5%	7,0%		7,5%	8,0%	8,5%	9,0%	9,5%	10,0%	10,5%	11,0%	11,5%	12,0%
0	81,177	66,569	55,473	46,933	40,277	35,020	30,818	27,415	24,629	22,320	20,387	18,751	17,355	16,151	15,106	14,190	13,383	12,667	12,027	11,453	10,935	10,466	10,038	9,647	9,288	0
1	80,463	66,133	55,214	46,789	40,205	34,996	30,822	27,438	24,662	22,359	20,429	18,795	17,398	16,194	15,147	14,230	13,422	12,704	12,063	11,487	10,968	10,497	10,068	9,676	9,316	1
2	79,822	65,473	54,769	46,487	39,999	34,854	30,724	27,369	24,614	22,325	20,405	18,778	17,386	16,185	15,141	14,226	13,418	12,701	12,061	11,486	10,967	10,496	10,068	9,676	9,316	2
3	78,496	64,808	54,316	46,177	39,786	34,706	30,621	27,297	24,563	22,289	20,379	18,759	17,372	16,175	15,133	14,220	13,414	12,698	12,059	11,484	10,966	10,496	10,067	9,675	9,315	3
4	77,507	64,136	53,857	45,962	39,567	34,554	30,514	27,122	24,509	22,250	20,351	18,738	17,357	16,154	15,125	14,214	13,409	12,695	12,056	11,482	10,964	10,494	10,066	9,674	9,314	4
5	76,516	63,459	53,382	45,540	39,343	34,396	30,403	27,142	24,452	22,209	20,321	18,716	17,340	16,151	15,115	14,206	13,404	12,690	12,052	11,479	10,962	10,492	10,064	9,673	9,314	5
6	75,522	62,776	52,920	45,211	39,113	34,234	30,287	27,060	24,392	22,165	20,289	18,692	17,322	16,137	15,105	14,198	13,397	12,685	12,048	11,476	10,959	10,490	10,062	9,671	9,312	6
7	74,528	62,090	52,443	44,878	38,878	34,068	30,168	26,974	24,330	22,120	20,254	18,666	17,303	16,122	15,081	14,179	13,390	12,679	12,043	11,472	10,955	10,484	10,060	9,669	9,311	7
8	73,533	61,399	51,961	44,539	38,639	33,897	30,045	26,885	24,265	22,071	20,219	18,639	17,282	16,107	15,081	14,179	13,382	12,673	12,038	11,467	10,952	10,484	10,057	9,667	9,309	8
9	72,538	60,706	51,474	44,195	38,394	33,721	29,919	26,792	24,197	22,021	20,181	18,611	17,260	16,090	15,067	14,169	13,373	12,666	12,032	11,463	10,948	10,480	10,054	9,664	9,306	9
10	71,543	60,008	50,983	43,846	38,145	33,542	29,788	26,697	24,126	21,969	20,141	18,581	17,237	16,072	15,053	14,157	13,364	12,658	12,026	11,457	10,943	10,476	10,051	9,661	9,304	10
11	70,548	59,308	50,488	43,492	37,890	33,358	29,654	26,598	24,053	21,914	20,100	18,549	17,213	16,052	15,038	14,145	13,354	12,650	12,019	11,452	10,938	10,472	10,047	9,658	9,301	11
12	69,553	58,604	49,985	43,133	37,631	33,169	29,516	26,497	23,977	21,857	20,057	18,516	17,187	16,032	15,022	14,132	13,344	12,641	12,012	11,445	10,933	10,468	10,043	9,655	9,298	12
13	68,560	57,898	49,480	42,769	37,367	32,977	29,375	26,391	23,899	21,797	20,011	18,481	17,160	16,011	15,005	14,118	13,332	12,632	12,004	11,439	10,927	10,463	10,039	9,651	9,294	13
14	67,568	57,189	48,970	42,401	37,099	32,780	29,229	26,283	23,817	21,736	19,964	18,445	17,131	15,988	14,987	14,104	13,321	12,622	11,996	11,432	10,921	10,458	10,034	9,647	9,291	14
15	66,579	56,479	48,458	42,028	36,827	32,580	29,081	26,172	23,734	21,672	19,916	18,407	17,102	15,965	14,968	14,089	13,308	12,612	11,987	11,425	10,915	10,452	10,030	9,643	9,287	15
16	65,592	55,768	47,942	41,653	36,551	32,376	28,929	26,059	23,648	21,607	19,866	18,368	17,072	15,941	14,949	14,073	13,296	12,602	11,979	11,417	10,909	10,447	10,025	9,639	9,283	16
17	64,610	55,057	47,425	41,274	36,272	32,169	28,775	25,943	23,561	21,540	19,814	18,328	17,041	15,917	14,930	14,058	13,283	12,591	11,970	11,410	10,903	10,442	10,021	9,635	9,280	17
18	63,632	54,346	46,905	40,892	35,990	31,959	28,618	25,825	23,471	21,472	19,768	18,288	17,009	15,892	14,910	14,042	13,270	12,581	11,962	11,403	10,897	10,437	10,016	9,631	9,277	18
19	62,657	53,634	46,383	40,507	35,704	31,746	28,458	25,704	23,379	21,402	19,708	18,246	16,976	15,866	14,899	14,026	13,257	12,570	11,953	11,396	10,891	10,432	10,012	9,627	9,274	19
20	61,686	52,922	45,868	40,118	35,415	31,530	28,295	25,581	23,286	21,330	19,652	18,203	16,945	15,836	14,869	14,009	13,244	12,560	11,944	11,389	10,885	10,427	10,008	9,624	9,271	20
21	60,717	52,208	45,330	39,725	35,121	31,309	28,128	25,454	23,189	21,256	19,595	18,159	16,908	15,812	14,847	13,992	13,230	12,549	11,936	11,382	10,879	10,422	10,004	9,621	9,268	21
22	59,749	51,492	44,797	39,327	34,822	31,083	27,967	25,323	23,088	21,179	19,535	18,112	16,871	15,784	14,824	13,974	13,216	12,537	11,926	11,374	10,873	10,417	10,000	9,617	9,265	22
23	58,780	50,771	44,258	38,922	34,516	30,852	27,788	25,188	22,984	21,098	19,472	18,062	16,832	15,753	14,800	13,964	13,200	12,524	11,916	11,366	10,866	10,411	9,995	9,613	9,262	23
24	57,809	50,045	43,713	38,511	34,204	30,613	27,598	25,047	22,875	21,013	19,406	18,010	16,791	15,720	14,773	13,953	13,183	12,510	11,904	11,366	10,858	10,405	9,990	9,610	9,258	24
25	56,837	49,315	43,162	38,092	33,885	30,369	27,409	24,901	22,761	20,924	19,335	17,954	16,747	15,684	14,745	13,910	13,164	12,495	11,891	11,345	10,849	10,397	9,983	9,603	9,253	25
26	55,864	48,580	42,604	37,666	33,559	30,117	27,215	24,750	22,643	20,830	19,261	17,896	16,699	15,646	14,714	13,885	13,143	12,478	11,877	11,334	10,840	10,389	9,976	9,597	9,248	26
27	54,890	47,840	42,040	37,234	33,226	29,859	27,014	24,593	22,519	20,732	19,184	17,833	16,649	15,606	14,681	13,858	13,121	12,459	11,862	11,321	10,829	10,380	9,968	9,591	9,243	27
28	53,915	47,097	41,470	36,795	32,886	29,595	26,807	24,430	22,390	20,630	19,102	17,768	16,596	15,562	14,645	13,828	13,097	12,439	11,845	11,307	10,817	10,369	9,960	9,583	9,236	28
29	52,940	46,349	40,893	36,349	32,539	29,324	26,594	24,261	22,256	20,523	19,016	17,698	16,540	15,516	14,608	13,797	13,071	12,417	11,827	11,291	10,803	10,358	9,950	9,575	9,229	29
30	51,965	45,597	40,312	35,896	32,185	29,045	26,374	24,087	22,117	20,411	18,926	17,625	16,480	15,467	14,567	13,763	13,042	12,393	11,807	11,274	10,789	10,346	9,939	9,565	9,220	30
31	50,990	44,842	39,724	35,437	31,824	28,761	26,148	23,906	21,973	20,295	18,831	17,548	16,417	15,415	14,524	13,727	13,012	12,368	11,785	11,255	10,773	10,332	9,927	9,555	9,211	31
32	50,015	44,083	39,131	34,971	31,456	28,469	25,915	23,720	21,822	20,173	18,732	17,467	16,350	15,360	14,478	13,689	12,980	12,340	11,761	11,235	10,756	10,317	9,914	9,543	9,201	32
33	49,041	43,322	38,532	34,499	31,082	28,170	25,676	23,528	21,667	20,046	18,628	17,381	16,279	15,301	14,429	13,647	12,945	12,311	11,736	11,214	10,737	10,301	9,900	9,531	9,190	33
34	48,067	42,557	37,929	34,020	30,700	27,865	25,431	23,329	21,505	19,915	18,520	17,292	16,205	15,239	14,377	13,604	12,908	12,279	11,709	11,190	10,717	10,283	9,884	9,517	9,178	34
35	47,095	41,789	37,320	33,535	30,312	27,553	25,178	23,124	21,338	19,777	18,407	17,198	16,127	15,174	14,321	13,557	12,868	12,245	11,680	11,165	10,695	10,264	9,868	9,502	9,165	35
36	46,124	41,019	36,707	33,044	29,917	27,234	24,920	22,913	21,165	19,635	18,289	17,100	16,045	15,104	14,263	13,507	12,825	12,208	11,648	11,138	10,671	10,243	9,849	9,486	9,151	36
37	45,155	40,246	36,088	32,547	29,516	26,908	24,654	22,695	20,986	19,486	18,166	16,997	15,958	15,031	14,201	13,454	12,780	12,169	11,614	11,108	10,645	10,220	9,829	9,469	9,135	37
38	44,187	39,471	35,465	32,043	29,107	26,575	24,381	22,471	20,800	19,332	18,037	16,899	15,867	14,954	14,135	13,398	12,732	12,128	11,578	11,076	10,617	10,196	9,808	9,449	9,118	38
39	43,221	38,694	34,836	31,533	28,692	26,235	24,101	22,240	20,608	19,172	17,903	16,876	15,772	14,873	14,066	13,338	12,680	12,083	11,539	11,042	10,587	10,169	9,784	9,428	9,099	39
40	42,256	37,914	34,204	31,018	28,269	25,888	23,815	22,002	20,410	19,006	17,763	16,668	15,671	14,787	13,992	13,275	12,625	12,035	11,497	11,006	10,555	10,140	9,759	9,406	9,079	40
41	41,295	37,134	33,567	30,496	27,84																					

Genaueres Alter in Jahren	Zinsfuß																Genaueres Alter in Jahren									
	0,0%	0,5%	1,0%	1,5%	2,0%	2,5%	3,0%	3,5%	4,0%	4,5%	5,0%	5,5%	6,0%	6,5%	7,0%	7,5%		8,0%	8,5%	9,0%	9,5%	10,0%	10,5%	11,0%	11,5%	12,0%
50	32,842	30,117	27,721	25,607	23,735	22,073	20,592	19,269	18,082	17,016	16,054	15,184	14,395	13,678	13,024	12,426	11,879	11,375	10,912	10,485	10,089	9,723	9,382	9,066	8,770	50
51	31,932	29,346	27,004	25,046	23,255	21,661	20,237	18,962	17,816	16,784	15,851	15,007	14,239	13,540	12,902	12,318	11,782	11,289	10,835	10,415	10,026	9,666	9,331	9,019	8,728	51
52	31,029	28,577	26,408	24,484	22,772	21,244	19,876	18,649	17,543	16,546	15,643	14,823	14,078	13,397	12,775	12,205	11,681	11,199	10,753	10,342	9,960	9,606	9,276	8,969	8,682	52
53	30,133	27,811	25,751	23,918	22,284	20,822	19,510	18,330	17,265	16,302	15,428	14,634	13,910	13,249	12,643	12,087	11,576	11,104	10,668	10,265	9,890	9,542	9,218	8,916	8,634	53
54	29,245	27,048	25,094	23,351	21,793	20,395	19,138	18,005	16,980	16,052	15,208	14,439	13,737	13,095	12,506	11,984	11,521	11,105	10,719	10,362	10,034	9,737	9,461	9,205	8,962	54
55	28,364	26,284	24,438	22,782	21,298	19,964	18,761	17,674	16,690	15,795	14,981	14,238	13,559	12,936	12,363	11,836	11,350	10,901	10,485	10,099	9,739	9,405	9,093	8,801	8,528	55
56	27,492	25,534	23,782	22,212	20,801	19,529	18,379	17,338	16,393	15,533	14,749	14,031	13,374	12,771	12,216	11,704	11,231	10,793	10,387	10,009	9,658	9,331	9,025	8,739	8,471	56
57	26,628	24,783	23,129	21,641	20,301	19,090	17,993	16,998	16,092	15,266	14,511	13,819	13,184	12,601	12,063	11,566	11,106	10,680	10,284	9,916	9,573	9,253	8,954	8,673	8,411	57
58	25,774	24,038	22,478	21,070	19,799	18,648	17,603	16,652	15,785	14,992	14,267	13,601	12,989	12,425	11,904	11,423	10,977	10,562	10,177	9,819	9,484	9,171	8,879	8,605	8,349	58
59	24,930	23,299	21,829	20,500	19,296	18,203	17,208	16,301	15,473	14,714	14,018	13,378	12,788	12,244	11,741	11,275	10,842	10,441	10,066	9,717	9,391	9,086	8,800	8,532	8,280	59
60	24,096	22,566	21,183	19,929	18,791	17,755	16,810	15,947	15,156	14,430	13,763	13,149	12,582	12,058	11,572	11,122	10,703	10,314	9,951	9,611	9,294	8,997	8,718	8,457	8,211	60
61	23,271	21,838	20,539	19,359	18,284	17,304	16,408	15,587	14,834	14,141	13,503	12,915	12,370	11,866	11,398	10,964	10,559	10,182	9,830	9,501	9,193	8,904	8,633	8,377	8,137	61
62	22,456	21,117	19,899	18,789	17,777	16,851	16,002	15,223	14,507	13,847	13,238	12,674	12,153	11,669	11,219	10,800	10,410	10,046	9,705	9,386	9,087	8,806	8,543	8,294	8,060	62
63	21,650	20,400	19,260	18,219	17,267	16,394	15,592	14,854	14,174	13,547	12,966	12,428	11,929	11,465	11,033	10,630	10,255	9,903	9,574	9,266	8,976	8,704	8,448	8,207	7,979	63
64	20,853	19,688	18,623	17,649	16,755	15,933	15,177	14,479	13,835	13,239	12,687	12,174	11,698	11,254	10,840	10,454	10,093	9,755	9,437	9,140	8,860	8,596	8,348	8,114	7,893	64
65	20,062	18,980	17,988	17,077	16,239	15,468	14,756	14,099	13,490	12,925	12,401	11,913	11,459	11,036	10,640	10,270	9,924	9,599	9,294	9,007	8,737	8,483	8,243	8,016	7,801	65
66	19,278	18,275	17,352	16,503	15,721	14,998	14,330	13,711	13,137	12,603	12,107	11,644	11,213	10,809	10,432	10,079	9,747	9,436	9,143	8,867	8,607	8,362	8,130	7,911	7,704	66
67	18,501	17,572	16,716	15,927	15,198	14,522	13,897	13,316	12,775	12,273	11,804	11,366	10,957	10,574	10,215	9,878	9,562	9,264	8,984	8,719	8,470	8,234	8,011	7,800	7,600	67
68	17,729	16,872	16,081	15,348	14,670	14,041	13,457	12,913	12,406	11,934	11,492	11,079	10,692	10,329	9,989	9,669	9,367	9,083	8,816	8,563	8,324	8,098	7,884	7,681	7,488	68
69	16,964	16,175	15,445	14,768	14,139	13,555	13,010	12,500	12,029	11,586	11,171	10,782	10,418	10,075	9,753	9,450	9,164	8,894	8,639	8,398	8,169	7,953	7,748	7,553	7,369	69
70	16,205	15,482	14,810	14,186	13,605	13,064	12,558	12,086	11,644	11,230	10,841	10,477	10,134	9,811	9,507	9,221	8,950	8,694	8,452	8,223	8,006	7,800	7,604	7,418	7,241	70
71	15,455	14,793	14,178	13,604	13,068	12,568	12,100	11,662	11,244	10,846	10,471	10,126	9,801	9,538	9,252	8,982	8,727	8,485	8,256	8,039	7,833	7,637	7,451	7,273	7,104	71
72	14,714	14,110	13,547	13,022	12,530	12,069	11,638	11,233	10,852	10,494	10,157	9,839	9,539	9,256	8,988	8,735	8,495	8,267	8,051	7,846	7,651	7,465	7,288	7,120	6,959	72
73	13,982	13,434	12,921	12,441	11,991	11,569	11,172	10,798	10,447	10,116	9,803	9,508	9,229	8,965	8,715	8,478	8,253	8,039	7,836	7,643	7,459	7,284	7,117	6,957	6,805	73
74	13,263	12,767	12,301	11,864	11,453	11,067	10,704	10,361	10,037	9,732	9,443	9,170	8,911	8,666	8,433	8,213	8,003	7,803	7,613	7,431	7,259	7,094	6,937	6,786	6,642	74
75	12,558	12,109	11,688	11,292	10,919	10,567	10,235	9,921	9,625	9,344	9,078	8,826	8,587	8,360	8,145	7,940	7,744	7,558	7,381	7,212	7,050	6,896	6,748	6,607	6,471	75
76	11,868	11,464	11,084	10,726	10,388	10,069	9,767	9,481	9,210	8,953	8,710	8,478	8,258	8,049	7,850	7,660	7,479	7,306	7,142	6,984	6,833	6,689	6,551	6,419	6,292	76
77	11,194	10,833	10,492	10,169	9,864	9,575	9,301	9,042	8,795	8,561	8,339	8,127	7,925	7,733	7,550	7,375	7,208	7,048	6,896	6,750	6,610	6,476	6,347	6,224	6,105	77
78	10,541	10,218	9,912	9,623	9,348	9,088	8,841	8,606	8,382	8,170	7,967	7,774	7,590	7,414	7,246	7,086	6,932	6,785	6,644	6,510	6,380	6,256	6,137	6,022	5,912	78
79	9,909	9,621	9,342	9,080	8,844	8,610	8,387	8,176	7,974	7,781	7,598	7,422	7,255	7,095	6,941	6,795	6,654	6,519	6,390	6,266	6,147	6,032	5,922	5,816	5,714	79
80	9,300	9,045	8,802	8,572	8,352	8,143	7,943	7,753	7,571	7,398	7,232	7,073	6,921	6,776	6,637	6,503	6,375	6,251	6,133	6,019	5,910	5,805	5,703	5,606	5,512	80
81	8,716	8,491	8,276	8,071	7,875	7,689	7,510	7,340	7,177	7,021	6,872	6,729	6,592	6,460	6,334	6,212	6,096	5,983	5,876	5,772	5,672	5,575	5,482	5,393	5,306	81
82	8,159	7,960	7,770	7,588	7,415	7,249	7,090	6,938	6,792	6,653	6,519	6,390	6,267	6,148	6,034	5,924	5,819	5,717	5,619	5,525	5,433	5,346	5,261	5,179	5,100	82
83	7,628	7,453	7,286	7,125	6,972	6,825	6,684	6,549	6,419	6,294	6,175	6,059	5,949	5,842	5,740	5,641	5,545	5,453	5,365	5,279	5,197	5,117	5,039	4,965	4,893	83
84	7,124	6,971	6,824	6,683	6,547	6,417	6,293	6,173	6,058	5,947	5,840	5,737	5,638	5,543	5,451	5,362	5,276	5,194	5,114	5,036	4,962	4,889	4,819	4,752	4,686	84
85	6,646	6,512	6,383	6,260	6,141	6,026	5,916	5,810	5,708	5,610	5,515	5,424	5,336	5,251	5,169	5,089	5,012	4,938	4,866	4,797	4,730	4,664	4,601	4,540	4,481	85
86	6,195	6,078	5,965	5,857	5,753	5,652	5,556	5,462	5,372	5,286	5,202	5,121	5,042	4,967	4,894	4,823	4,754	4,688	4,624	4,561	4,501	4,442	4,386	4,330	4,277	86
87	5,769	5,667	5,569	5,475	5,384	5,296	5,211	5,129	5,050	4,973	4,899	4,828	4,759	4,692	4,627	4,564	4,503	4,444	4,386	4,331	4,277	4,224	4,173	4,124	4,076	87
88	5,371	5,282	5,197	5,115	5,036	4,959	4,885	4,813	4,743	4,676	4,611	4,548	4,487	4,428	4,370	4,314	4,260	4,208	4,157	4,107	4,059	4,012	3,967	3,922	3,879	88
89	5,000	4,923	4,849	4,778	4,709	4,642	4,577	4,514	4,453	4,394	4,337	4,282	4,228	4,176	4,125	4,076	4,028	3,981	3,936	3,892	3,849	3,808	3,767	3,727	3,689	89
90	4,656	4,590	4,526	4,464	4,403	4,345	4,289	4,234	4,181	4,129	4,079	4,030	3,983	3,937	3,892	3,849	3,807	3,765	3,725	3,686	3,648	3,611	3,575	3,540	3,506	90
91	4,339	4,282	4,226	4,172	4,120	4,069	4,020	3,972	3,926	3,881	3,837	3,794	3,753	3,712	3,673	3,635	3,597	3,561	3,526	3,491	3,457	3,425	3,393	3,361	3,331	91
92	4,047	3,998	3,950	3,903	3,858	3,814	3,771	3,729	3,689	3,649	3,611	3,573	3,537	3,501	3,467	3,433	3,400	3,368	3,337	3,307	3,277	3,248	3,219	3,192	3,165	92
93	3,780	3,737	3,696	3,655	3,616	3,577	3																			