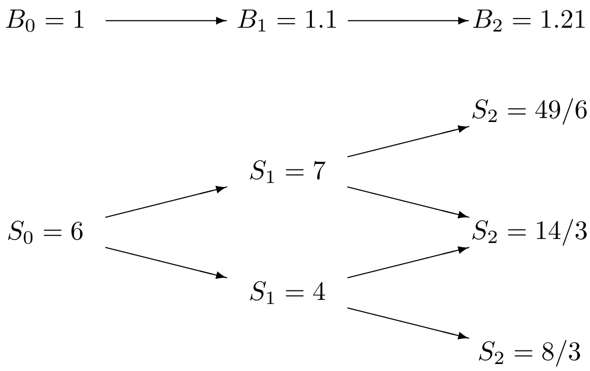
Consider the following two-step Cox-Ross-Rubinstein model ($a = -1/3$, $r = 1/10$, $b = 1/6$) for a financial market with bond B and stock S:



Assume furthermore that $P(\{\omega_i\}) = 0.2$, $1 \leq i \leq 3$ and $P(\{\omega_4\}) = 0.4$, where the 4-th trajectory is the one going twice up.

(a) Compute the set of equivalent martingale measures $\mathcal{M}^e(\tilde{S})$ and the Radon-Nikodym derivative of the martingale measure Q with respect to P. (1)

(b) Compute the arbitrage-free price at time $N = 0$ of a claim with payoff

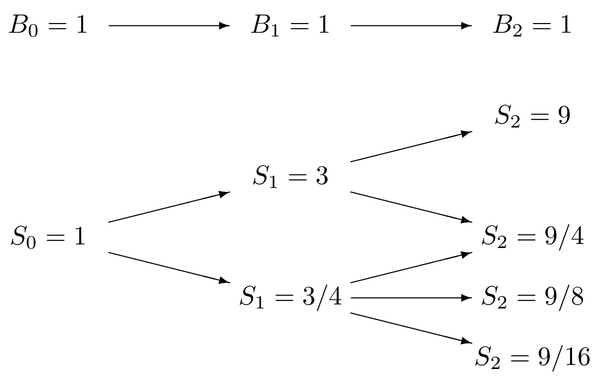
$$\big(\, 61/6 \, , \, 61/6 \, , \, 14/3 \, , \, 0 \, \big)$$

and maturity $N = 2$. (1)

(c) Compute the replicating strategy of the claim introduced in part (b). (1)

(d) Compute your expected total gain (or loss) if you buy the claim of (b) at time $N = 0$, i.e. compute the difference of the expected outcome of the claim (with respect to the measure P) and the invested premium. This difference is non-vanishing: is this in contradiction to no-arbitrage arguments? (1)

Consider a two-step model for a financial market with bond B and stock S. The bond B is assumed to be constant, i.e. $B_n = 1$ for $n = 0, 1, 2$, the stock S is assumed to move according to the following tree:



Assume furthermore that $P(\{\omega_i\}) = 0.2$, $1 \leq i \leq 5$.

(d) Compute the set of equivalent martingale measures $\mathcal{M}^e(\tilde{S})$ as well as the set of absolutely continuous martingale measures $\mathcal{M}^a(\tilde{S})$. (1)

(e) Compute the arbitrage-free price(s) at times $n = 0, 1$ of a European put option $(S_2 - K)_{-}$ with strike $K = 9/2$ and maturity $N = 2$. (1)

(f) How to replicate the European digital option from (e). (1)

TASK 2

Fix $T > 0$. Consider a Bachelier Model with constant interest rate $r = 0$ and stock price S, i.e. under the martingale measure Q the stock price process is given by

$$S_t = S_0 + \sigma B_t, \quad S_0, \sigma > 0, \quad 0 \leq t \leq T.$$

Here, B denotes a standard Brownian motion under the martingale measure Q.

(a) Compute the arbitrage-free price v of an option with payoff

$$S_T^2.$$

Hint: consider $S_T = (S_T - S_0) + S_0$ to make the computation easier. (2)

(b) The price v depends on T, S_0, σ. Compute the Delta of the price v, i.e. the partial derivative of the price with respect to S_0 and argue how one can construct the Hedging strategy of S_T^2 with the Delta v. (2)

TASK 3

Consider a Black-Scholes Model with constant interest rate $r = 0$ and stock price S, i.e. under the martingale measure Q the stock price process is given by

$$S_t = S_0 \exp(-\frac{\sigma^2 t}{2} + \sigma B_t), \quad S_0, \sigma > 0, \quad 0 \leq t \leq T.$$

Here, B denotes a standard Brownian motion under the martingale measure Q.

(a) Compute the arbitrage-free price $p(S_0, T)$ of a European "square" option

$$(S_T - K)^2,$$

Apply that $(S_t)_{0 \leq t \leq T}$ is a martingale with $S_0 = 1$. (2)

(b) Try to guess a replication strategy from the well-known result

$$S_t = S_0 + \int_0^t \sigma S_s dB_s,$$

for $0 \leq t \leq T$ and the observation that S_t^2 is up to a factor equal to $\exp(-\frac{\eta^2 t}{2} + \eta B_t)$ for another choice of volatility $\eta > 0$. (2)