TASK 1

Consider the following two-step Cox-Ross-Rubinstein model ($a = -1/5$, $r = 1$, $b = 3/2$) for a financial market with bond B and stock S. Assume furthermore that $P(\{\omega_i\}) = 0.25$, $1 \leq i \leq 4$.

(a) Compute the set of equivalent martingale measures $\mathcal{M}^e(\tilde{S})$ and the Radon-Nikodym derivative of the martingale measure Q with respect to P. (1)

(b) Compute the arbitrage-free price at time $N = 0$ of a claim with payoff

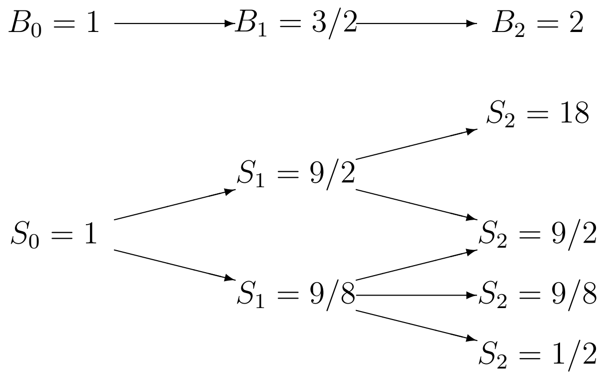
$$\left(4\ ,\ 3\ ,\ 0\ ,\ 0 \right)$$

and maturity $N = 2$. It this claim of European type, i.e. does it only depend on the price at time $N = 2$? (1)

(c) Compute the replicating strategy of the claim introduced in part (b). (1)

(d) Compute your expected total gain (or loss) if you buy the claim of (b) at time $N = 0$, i.e. compute the difference of the expected outcome of the claim (with respect to the measure P) and the invested premium. Is this in contradiction to no-arbitrage arguments? (1)

Consider a two-step model for a financial market with bond B and stock S. The bond B and the stock S are assumed to move according to the following tree:



Assume furthermore that $P(\{\omega_i\}) = 0.2$, $1 \leq i \leq 5$.

(e) Compute the set of equivalent martingale measures $\mathcal{M}^e(\tilde{S})$ as well as the set of absolutely continuous martingale measures $\mathcal{M}^a(\tilde{S})$. (1)

(f) Compute the arbitrage-free price(s) at times $n = 0, 1$ of a European put option with strike $K = 9/4$ and maturity $N = 2$. Apply the put-call parity to calculate the prices of a European Call option. (1)

TASK 2

Consider a Bachelier Model with constant interest rate $r = 0$ and stock price S, i.e. under the martingale measure Q the stock price process is given by

$$S_t = S_0 + \sigma B_t, \quad S_0, \sigma > 0, \quad 0 \leq t \leq T.$$

Here, B denotes a standard Brownian motion under the martingale measure Q.

(a) Compute the arbitrage-free price v of an option with payoff

$$1_{S_T \geq S_0},$$

i.e. which pays one if $S_T \geq S_0$ and 0 otherwise. (2)

(b) Find a replicating strategy $(\phi_t)_{0 \leq t \leq T}$ for the option, i.e. solve the equation

$$1_{S_T \geq S_0} = v + \int_0^T \phi_s dS_s.$$

Write down the Hedging portfolio for the option and calculate the amount of money at each time in the Bank account. Recall that the derivative of the price v for time to maturity $T - t$ at the actual price S_t yields the hedging portfolio. (2)

TASK 3

Consider a Black-Scholes Model with constant interest rate $r = 0$ and stock price S, i.e. under the martingale measure Q the stock price process is given by

$$S_t = S_0 \exp(-\frac{\sigma^2 t}{2} + \sigma B_t), \quad S_0, \sigma > 0, \quad 0 \leq t \leq T.$$

Here, B denotes a standard Brownian motion under the martingale measure Q.

(a) Compute the arbitrage-free price p of the scaled European put option with payoff

$$(0.01 S_T - K)_-.$$

How does the put-call parity look for this type of option? (2)

(b) Calculate actual prices for $S_0 = 100, K = 1, \sigma = 0.2, T = 0,5$ and the actual hedging portfolio in this case. (2)