Name:

Mat.Nr.:

Bitte keinen Rotstift verwenden!

Finanzmathematik 1: diskrete Modelle (Vorlesungsprüfung) 24. Juni 2016 Stefan Gerhold

(Dauer 90 Minuten, Erlaubte Hilfsmittel: Schreibutensilien, nicht programmierbarer Taschenrechner, 1 selbstbeschriebenes A4 Blatt (beidseitig))

Anmeldung zur mündlichen Prüfung nach Absprache.

Bsp.	Max.	Punkte
1	7	
2	19	
3	6	
$\sum$	32	

1. Consider the following three one-period models.

   (A) $\Omega = \{\omega_1, \omega_2\}$, $r = \frac{1}{2}$,

   $$S_0 = 5, \quad S_1(\omega_1) = \frac{5}{2}, \quad S_1(\omega_2) = \frac{13}{2}.$$

   (B) $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $r = \frac{1}{3}$,

   $$S_0 = 4, \quad S_1(\omega_1) = \frac{28}{3}, \quad S_1(\omega_2) = 8, \quad S_1(\omega_3) = 4.$$

   (C) $\Omega = \{\omega_1, \omega_2, \omega_3\}$, $r = 0$, dimension $d = 2$,

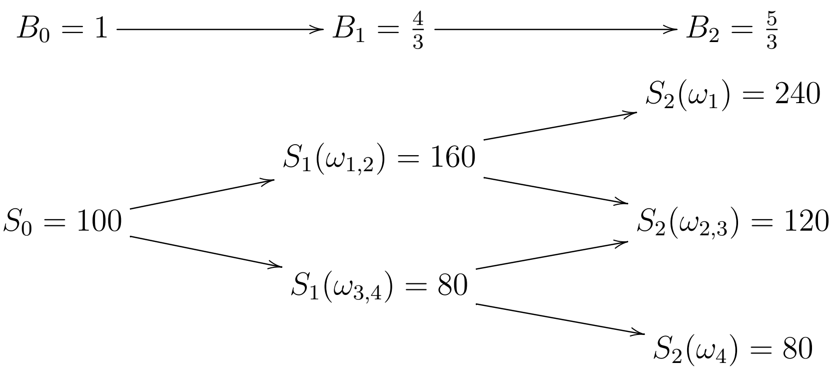
   $$S_0 = \begin{pmatrix} 10 \\ 10 \end{pmatrix}, \quad S_1(\omega_1) = \begin{pmatrix} 15 \\ 10 \end{pmatrix}, \quad S_1(\omega_2) = \begin{pmatrix} 10 \\ 15 \end{pmatrix}, \quad S_1(\omega_3) = \begin{pmatrix} 6 \\ 10 \end{pmatrix}.$$

   For each model, a probability measure is given that has positive mass on each scenario.

   (i) For each model, compute the set of equivalent martingale measures. If there is an arbitrage, give an arbitrage portfolio. (4 pts.)

   (ii) For the arbitrage free models, find the set of arbitrage free prices of a put option on S (or S^1, in case (C) is arbitrage free) with maturity 1 and strike $K = 10$. (3 pts.)

2. Consider the following two-period model with a risky asset S and a riskless asset B, on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. The filtration $(\mathcal{F}_t)_{t=0,1,2}$ is generated by S, i.e., $\mathcal{F}_t = \sigma(S_0, \ldots, S_t)$ for $t = 0, 1, 2$.



   (i) Explain what this diagram means, i.e., define $(\mathcal{F}_t)_{t=0,1,2}$, the process S and (a reasonable choice of) the measure $\mathbb{P}$ in standard mathematical notation. (3 pts.)

   (ii) Find all equivalent martingale measures $\mathbb{P}^*$. (Identify $\mathbb{P}^*$ with $(p_1, p_2, p_3, p_4) \in \mathbb{R}^4$; indicate the meaning of this identification.) (3 pts.)

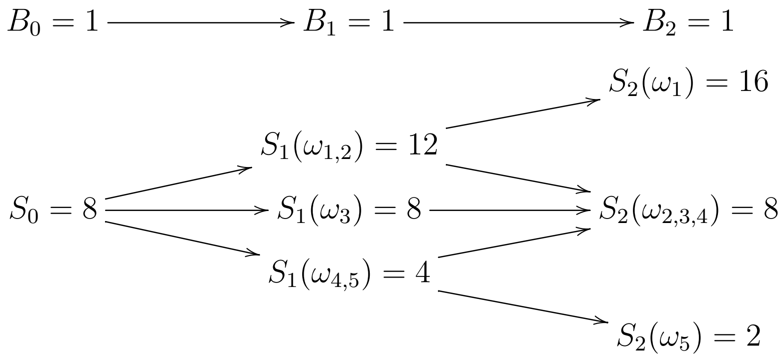
   (iii) Consider a European put option with maturity 2 and strike $K = 100$. Is it replicable? If so, find a replication strategy $(\overline{\xi}_t)_{t=1,2}$. (3 pts.)

(iv) Let $(V_t)_{t=0,1,2}$ be the discounted value process of this strategy. How is it defined? Denote by P the $\mathbb{P}^*$-law of the random variable V_1 (German: Verteilung der Zufallsvariable V_1 bezgl. $\mathbb{P}^*$). Then P is a map from where to where, and how is it defined? (2 pts.)

(v) Consider an American put with strike $K = 100$. What is the minimal capital needed at time zero to superhedge the put? What is its set of arbitrage free prices? (4 pts.)

(vi) State the definition of the stopping times $\tau_{\min}$ and $\tau_{\max}$ and compute them (for this put option). (4 pts.)

3. Consider the following two-period model, with $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, $\mathcal{F}_0 = \{\emptyset, \Omega\}$, $\mathcal{F}_1 = \sigma(S_1)$, $\mathcal{F}_2 = \sigma(S_1, S_2)$ and $\mathbb{P}(\{\omega_i\}) > 0$ for $i = 1, ..., 5$.



(i) Compute the set of equivalent martingale measures. Is the model free of arbitrage? Is it complete? (3 pts.)

(ii) Consider an American put with strike $K = 10$. Consider the boundary points $\pi_{\inf}$ and $\pi_{\sup}$ of the interval of arbitrage free prices. How can they be computed? (You do not need to compute their values in this example, just state the general result precisely.) What can you say about $\pi_{\inf}$ and/or $\pi_{\sup}$ in this example, if you consider only *deterministic* stopping times? (3 pts.)