Modelling the Claims Development Result for Solvency Purposes

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Collaboration

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Overview

1. Non-Life (NL) Insurance and Claims Reserving
2. Claims Development Result and Solvency
3. Bayesian Chain-Ladder Method
4. Conclusions

1. Non-Life (NL) Insurance and Claims Reserving

NL insurance company: **accounting year** $I + 1 = 2009$

budget statement at 1/1/2009
profit & loss (P&L) statement at 31/12/2009

	Budget 1/1/2009	P&L 31/12/2009
premium earned	4’000’000	4’020’000
claims incurred current accident year	-3’300’000	-3’340’000
loss experience prior accident years	**0**	**-40’000**
administrative expenses	-1’000’000	-1’090’000
investment income	500’000	510’000
income before taxes	200’000	60’000

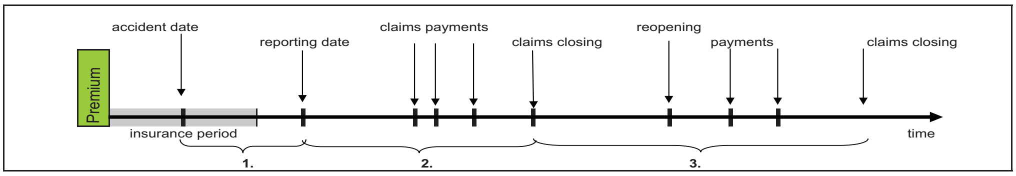
Questions and Terminology

- What is the position “**loss experience prior accident years**”?
- Why do we predict it by 0?
- What is the **uncertainty in this prediction**?

- What are (best estimate) **claims reserves**?
- Long term view versus the short term view

These questions can not be answered with simple concepts.

NL Claims Settlement Process



Often it takes **several years** until a claim is finally settled. Reasons:

1. **Reporting delay**: time lag between accident date and reporting date (notification at insurance company)
2. **Settlement delay**: time interval between reporting date and final settlement (severity of claim, recovery process, court decisions, etc.)
3. **Reopenings** due to new (unexpected) claim developments

Conclusions: Claims Reserving

- Every claim generates a (random) future payment cashflow.
- The claims reserves should suffice to meet this future cashflow

$\Longrightarrow$ **claims reserving is a prediction problem**.

- Determine the **prediction uncertainty**:

deterministic claims reserves $\Longleftrightarrow$ **stochastic claims payments**

Prediction Uncertainty

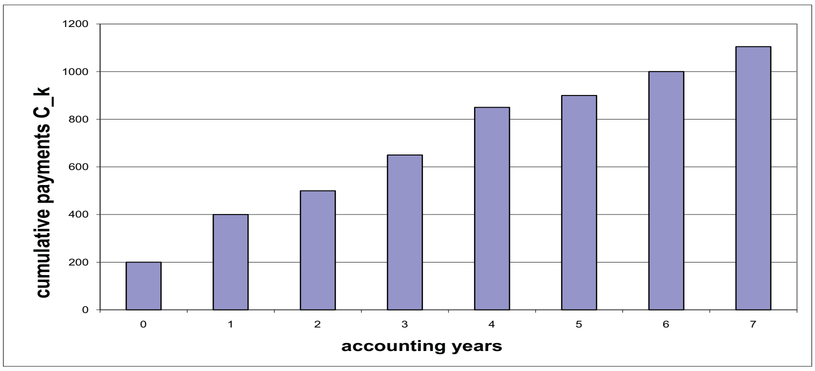
- X future cashflow (**random variable**) to be **predicted**.
- $\mathcal{D}$ information available at time I.
- Assume $\widehat{X}$ is a $\mathcal{D}$-measurable predictor for X.

The (conditional) **mean square error of prediction (MSEP)** is defined by

$$\mathrm{msep}_{X|\mathcal{D}}\left(\widehat{X}\right) = E\left[\left(X - \widehat{X}\right)^2 \middle| \mathcal{D}\right].$$

MSEP is the most common uncertainty measure in practice.

2. Claims Development Result and Solvency



- **incremental payments** in accounting year $k \geq 0$ are denoted by X_k
- **cumulative payments** are denoted by $C_j = \sum_{k=0}^{j} X_k$
- **ultimate (total) claim** is denoted by C_J
- **observations at time** k are denoted by $\mathcal{D}_k = \{C_j : \ j \leq k\}$

Ultimate Claims Prediction

Goal: Predict the ultimate claim C_J based on $\mathcal{D}_k$, $k \leq J$.

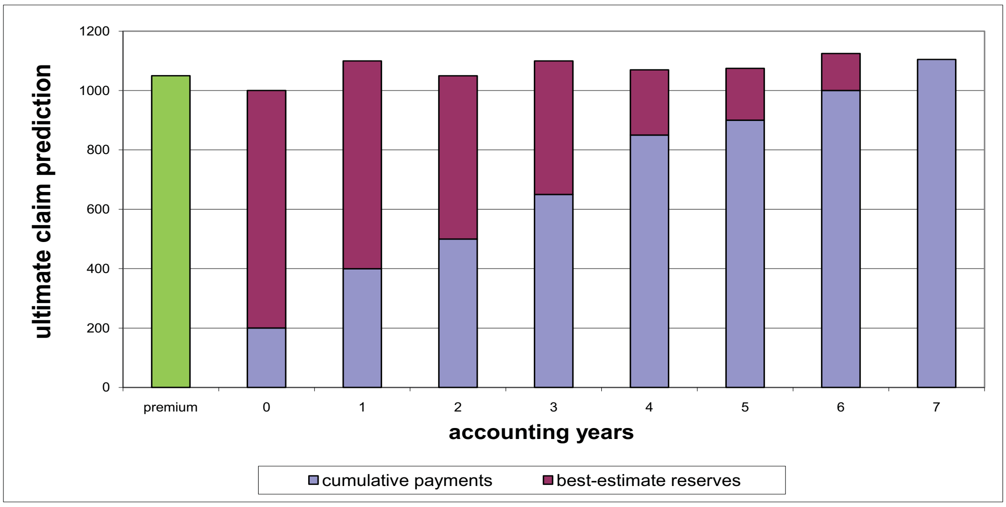
At time k, we use the (minimum variance) predictor

$$\widehat{C}_J^{(k)} = E\left[C_J \middle| \mathcal{D}_k \right].$$

First Consequences:

- $\widehat{C}_J^{(k)}$ minimizes the conditional MSEP.
- $\widehat{R}^{(k)} = \widehat{C}_J^{(k)} - C_k$ are the "**best-estimate reserves**" at time k.
- $\widehat{C}_J^{(0)}, \widehat{C}_J^{(1)}, \widehat{C}_J^{(2)}, \ldots$ is a **martingale**.

Time Series Ultimate Claims Prediction



Classical Uncertainty View

$\widehat{C}_J^{(k)}$ minimizes the conditional MSEP:

$$\mathrm{msep}_{C_J|\mathcal{D}_k}\left(\widehat{C}_J^{(k)}\right) = E\left[\left(C_J - \widehat{C}_J^{(k)}\right)^2 \middle| \mathcal{D}_k\right] = \mathrm{Var}\left(C_J \middle| \mathcal{D}_k\right).$$

Henceforth, we obtain the **prediction variance**.

- **Classical long term view:** Study this prediction variance:

E.g. Mack (1993), England-Verrall (2002), W.-Merz (2008).

- This is **not** the **Solvency View** **(short term view)** for the P&L position “loss experience prior accident years”.

P&L Position "Loss Experience Prior Accident Years"

The **Claims Development Result (CDR)** at time k is given by

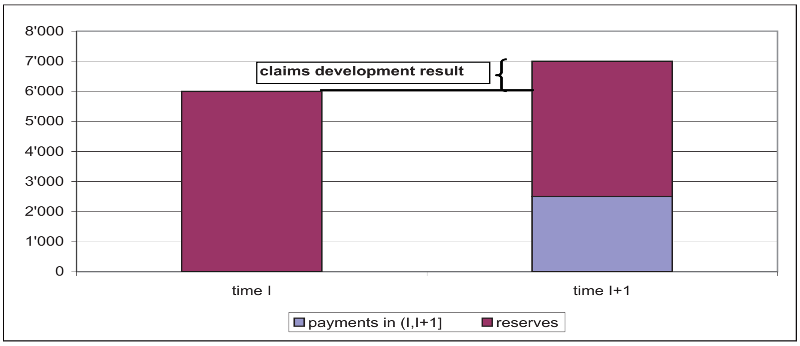
$$\mathrm{CDR}(k) = \widehat{C}_J^{(k-1)} - \widehat{C}_J^{(k)}. \qquad (1)$$

- This is the incorporation of the latest information $\mathcal{D}_k$ available at time k, i.e. update of information $\mathcal{D}_{k-1} \mapsto \mathcal{D}_k$.
- CDR exactly corresponds to "loss experience prior accident years".
- Under **Solvency 2** we need to study the **uncertainty** in this position:

  Question: Do we have sufficient provisions at time k to cover possible shortfalls in CDR?

Time Series of Claims Development Results

Short term view: changes over the next accounting year:



Therefore: study the **Claims Development Result (CDR)**.

Properties of Claims Development Results

We have (martingale property)

$$E\left[\mathrm{CDR}(k) \middle| \mathcal{D}_{k-1}\right] = 0. \tag{2}$$

Moreover, this implies

$\mathrm{CDR}(1), \mathrm{CDR}(2), \ldots$ are **uncorrelated** (not independent).

This immediately implies that

$$\begin{aligned} \mathrm{msep}_{C_J|\mathcal{D}_0}\left(\widehat{C}_J^{(0)}\right) &= \mathrm{Var}\left(C_J \middle| \mathcal{D}_0\right) \qquad (3) \\ &= \mathrm{Var}\left(\sum_{k\geq 1} \mathrm{CDR}(k) \middle| \mathcal{D}_0\right) = \sum_{k \geq 1} \mathrm{Var}\left(\mathrm{CDR}(k) \middle| \mathcal{D}_0\right). \end{aligned}$$

Budget and P&L Statement

Equation (2) implies prediction 0 of the CDR:

	Budget 1/1/2009	P&L 31/12/2009
premium earned	4'000'000	4'020'000
claims incurred current accident year	-3'300'000	-3'340'000
loss experience prior accident years	**0**	**-40'000**
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Prediction Uncertainty in accounting year k: study

$$\mathrm{msep}_{\mathrm{CDR}(k)|\mathcal{D}_{k-1}}(0) = \mathrm{Var}\left(\mathrm{CDR}(k) \middle| \mathcal{D}_{k-1}\right).$$

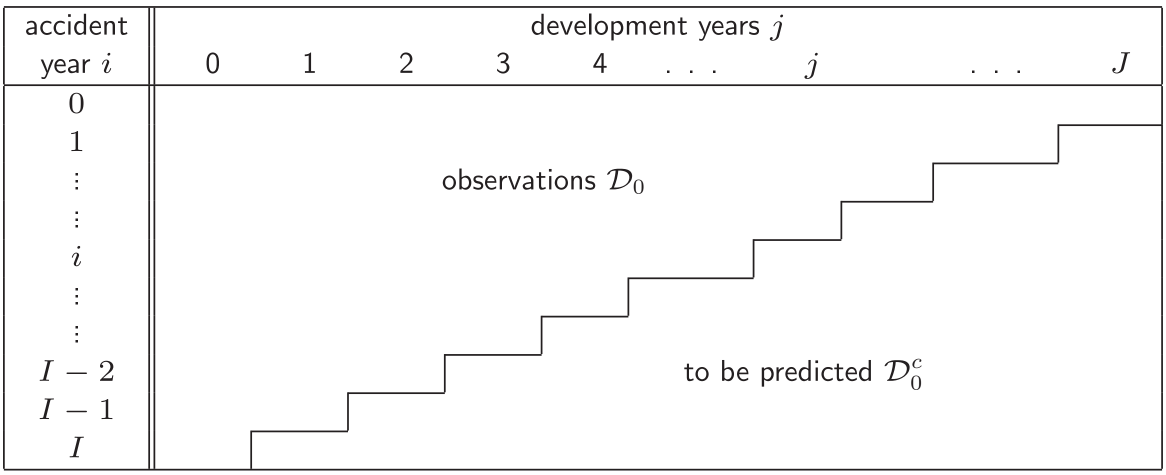
3. Bayesian Chain-Ladder Method

We use the following notation:

- **accident years** are denoted by $i \in \{0, \ldots, I\}$
- **development years** are denoted by $j \in \{0, \ldots, J\}$
- **accounting years** are given by $i + j = k$ (constant)
- **incremental payments** are denoted by $X_{i,j}$
- **cumulative payments** are denoted by

$$C_{i,j} = \sum_{l=0}^{j} X_{i,l}.$$

Loss Development Triangle at Time I



- **observations:** $\mathcal{D}_k = \{C_{i,j} : \; i + j \leq I + k\}$
- **to be predicted:** $\mathcal{D}_k^c = \{C_{i,j} : \; i + j > I + k, \; i \leq I\}$

Example 1: Cumulative Payments

	0	1	2	3	4	5	6	7	8	9
0	5'946975	9'668212	10'563929	10'771690	10'978394	11'040518	11'106331	11'121181	11'132310	11'148124
1	6'346756	9'593162	10'316383	10'468180	10'536004	10'572608	10'625360	10'636546	10'648192	
2	6'269090	9'245313	10'092366	10'355134	10'507837	10'573282	10'626827	10'635751		
3	5'863015	8'546239	9'268771	9'459424	9'592399	9'680740	9'724068			
4	5'778885	8'524114	9'178009	9'451404	9'681692	9'786916				
5	6'184793	9'013132	9'585897	9'830796	9'935753					
6	5'600184	8'493391	9'056505	9'282022						
7	5'288066	7'728169	8'256211							
8	5'290793	7'648729								
9	5'675568									

Observed historical **cumulative payments** at time $I = 9$

$$\mathcal{D}_0 = \{C_{i,j} : \ i + j \leq 9\}.$$

Bayesian CL Model Assumptions

- Conditional on $\mathbf{F} = (F_0, \ldots, F_{J-1})$ we have
  - ⋆ different accident years i are independent;
  - ⋆ $\{C_{i,j}\}_{j \geq 0}$ is a Markov process with

$$E\left[C_{i,j} \middle| C_{i,j-1}, \mathbf{F}\right] = F_{j-1}\, C_{i,j-1}, \qquad \text{for all } i, j.$$
$$\text{Var}\left(C_{i,j} \middle| C_{i,j-1}, \mathbf{F}\right) = \sigma^2_{j-1}(F_{j-1})\, C_{i,j-1}, \qquad \text{for all } i, j.$$

- The components of $\mathbf{F}$ are independent.

Note that the CL factors $\mathbf{F}$ are part of the model which allows for **Bayesian inference** and **parameter uncertainty study within the model**.

Properties of the Bayesian CL Model

Theorem 1. The posteriors of $\mathbf{F}$, given $\mathcal{D}_k$, are **independent**.

Theorem 2. The minimum variance predictor for $C_{i,J}$ is given by

$$\widehat{C}_{i,J}^{(k)} = E\left[C_{i,J}\middle|\,\mathcal{D}_k\right] = C_{i,I-i+k} \prod_{j=I-i+k}^{J-1} E\left[F_j\middle|\,\mathcal{D}_k\right].$$

Theorem 3. The prediction uncertainties $\mathrm{msep}_{C_{i,J}|\mathcal{D}_0}\left(\widehat{C}_{i,J}^{(0)}\right)$ and $\mathrm{msep}_{\mathrm{CDR}_i(1)|\mathcal{D}_0}(0)$ can be calculated analytically.

See Gisler-W. (2008) and Bühlmann et al. (2009).

Estimation of CL Factors

We choose a non-informative prior distribution for $\mathbf{F}$.

In that case we obtain at time k

$$\widehat{f}_j^{(k)} = E\left[F_j\middle|\,\mathcal{D}_k\right] = \frac{\sum_{i=0}^{(I-j-1+k)\wedge I} C_{i,j+1}}{\sum_{i=0}^{(I-j-1+k)\wedge I} C_{i,j}}.$$

Henceforth, the **CL predictor** for $C_{i,J}$ at time k is given by

$$\widehat{C}_{i,J}^{(k)} = E\left[C_{i,J}\middle|\,\mathcal{D}_k\right] = C_{i,I-i+k} \prod_{j=I-i+k}^{J-1} \widehat{f}_j^{(k)}.$$

Prediction Uncertainty (Linear Approximation)

$$\widehat{\mathrm{msep}}_{C_{i,J}|\mathcal{D}_0}\left(\widehat{C}_{i,J}^{(0)}\right) = \left(\widehat{C}_{i,J}^{(0)}\right)^2$$
$$\times \left[\sum_{j=I-i}^{J-1} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^{(0)}\right)^2}{\widehat{C}_{i,j}^{(0)}} + \sum_{j=I-i}^{J-1} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^{(0)}\right)^2}{\sum_{l=0}^{I-j-1} C_{l,j}}\right],$$

$$\widehat{\mathrm{msep}}_{\mathrm{CDR}_i(1)|\mathcal{D}_0}(0) = \left(\widehat{C}_{i,J}^{(0)}\right)^2$$
$$\times \left[\frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^{(0)}\right)^2}{\widehat{C}_{i,I-i}^{(0)}} + \frac{\widehat{\sigma}_{I-i}^2 / \left(\widehat{f}_{I-i}^{(0)}\right)^2}{\sum_{k=0}^{i-1} C_{k,j}} + \sum_{j=I-i+1}^{J-1} \frac{C_{I-j,j}}{\sum_{l=0}^{I-j} C_{l,j}} \frac{\widehat{\sigma}_j^2 / \left(\widehat{f}_j^{(0)}\right)^2}{\sum_{l=0}^{I-j-1} C_{l,j}}\right].$$

See Gisler-W. (2008), Merz-W. (2008) and Bühlmann et al. (2009).

Example 1, revisited

i	CL reserves $\widehat{R}_i^{(0)}$	$\widehat{\mathrm{msep}}_{\mathrm{CDR}_i(1)\|\mathcal{D}_0}(0)^{1/2}$		$\widehat{\mathrm{msep}}_{C_{i,J}\|\mathcal{D}_0}\left(\widehat{C}_{i,J}^{(0)}\right)^{1/2}$	
1	15'126	267	1.8%	267	1.8%
2	26'257	884	3.4%	914	3.5%
3	34'538	2'948	8.5%	3'058	8.9%
4	85'302	7'018	8.2%	7'628	8.9%
5	156'494	32'470	20.7%	33'341	21.3%
6	286'121	66'178	23.1%	73'467	25.7%
7	449'167	50'296	11.2%	85'398	19.0%
8	1'043'242	104'311	10.0%	134'337	12.9%
9	3'950'815	385'773	9.8%	410'817	10.4%
cov.		94'134		116'810	
Total	6'047'061	420'220	6.9%	462'960	7.7%

We see that the ratio is around 90%.

More Examples

	CL reserves $\widehat{R}^{(0)}$	$\widehat{\mathrm{msep}}_{\mathrm{CDR}(1)\mid\mathcal{D}_0}(0)^{1/2}$		$\widehat{\mathrm{msep}}_{C_J\mid\mathcal{D}_0}\left(\widehat{C}_J^{(0)}\right)^{1/2}$	
Example 2 (commercial liability)	646'496	19'300	3.0%	31'344	4.8%
Example 3 (Merz-W. (2008))	2'237'826	81'080	3.6%	108'401	4.8%

We see that the ratio is around 60% in Example 2.

We see that the ratio is around 75% in Example 3.

Example: Italian MTPL (37 Companies)

company	business volume	msep total runoff (in % reserves)	msep CDR(1) (in % reserves)	$\frac{\text{msep CDR(1)}}{\text{msep total runoff}}$ (in %)
1	100.0	4.03	3.24	80.4
2	100.0	2.90	2.36	81.4
3	100.0	2.41	1.98	82.3
4	100.0	3.45	2.85	82.6
5	61.8	3.66	3.04	82.9
6	56.9	5.54	4.50	81.2
7	53.0	4.52	3.70	81.8
8	49.4	4.60	3.82	83.1
9	46.2	5.61	4.59	81.8
10	41.6	5.32	4.36	82.0
⋮		⋮	⋮	⋮
30	3.5	18.02	14.78	82.0
31	3.4	17.23	13.92	80.8
32	2.6	18.73	14.89	79.5
33	2.5	23.11	19.10	82.6
34	2.2	20.83	17.53	84.2
35	2.0	17.01	13.87	81.5
36	1.8	26.16	21.54	82.4
37	1.8	27.79	22.25	80.1
Total		0.96	0.78	81.8

For more explanation see Bühlmann et al. (2009).

4. Conclusions

- In all examples considered: the ratio between one-year CDR risk and full run-off risk was within the intervall $[50\%, 95\%]$ (range between liability insurance and property insurance).

  This is also supported by the AISAM-ACME field study 2007.

- We have measured risk with the help of the conditional MSEP. For Value-at-Risk or Expected Shortfall considerations fit distribution with appropriate moments ($=$ proxy).

- A **full distributional approach** can only be solved **numerically**, e.g. Markov chain Monte Carlo (MCMC) methods (Bayesian models).

Conclusions

- Dependence is not appropriately modelled. Especially, **accounting year dependence and claims inflation** needs special care (MCMC methods).

- The one-year CDR view needs a **Cost-of-Capital charge (risk margin) for the risk that is beyond the one-year time horizon**, **market-value margin**, see Salzmann-W. (2009).

- **Discounting and financial risk** is not considered. First results on this topic are obtained in W.-Bühlmann (2009).

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