A NONSYMMETRIC VERSION OF MEYER'S INEQUALITY IN INFINITE DIMENSION

JAN MAAS (TU DELFT)

A classical result in the theory of Malliavin calculus due to P.A. Meyer states that

$$\|\phi\|_p + \|D\phi\|_p \cong \|(-\mathcal{L})^{1/2}\phi\|_p, \quad 1 < p < \infty.$$

In this formula $\mathcal{L}$ denotes the classical Ornstein-Uhlenbeck operator and D is the Malliavin derivative.

We consider the transition semigroup of the following stochastic differential equation in a Banach space E,

$$\begin{cases} dX(t) &= AX(t)dt + dW_H(t) \\ X(0) &= x \end{cases}$$

where A is the generator of a C_0-semigroup in E, $H \subset E$ is a Hilbert space, W_H is an H-cylindrical Wiener process and $x \in E$. Assuming the existence of a solution $X(t,x)$ and an invariant measure μ, the corresponding transition semigroup is defined by

$$P(t)f(x) := \mathbb{E}(f(X(t,x))), \quad f \in C_b(E),$$

and extends to a C_0-semigroup on $L^p(E,\mu)$ for $1 \leq p < \infty$. Its generator will be denoted by L and the Fréchet derivative in the direction of H is called D_H. We will show that the following inequalities hold,

$$\|\phi\|_p + \|D_H\phi\|_p \lesssim \|\phi\|_p + \|(-L)^{1/2}\phi\|_p, \quad 1 < p \leq 2,$$

and

$$\|\phi\|_p + \|(-L)^{1/2}\|_p \lesssim \|\phi\|_p + \|D_H\phi\|_p, \quad 2 \leq p < \infty.$$

This is joint work with Jan van Neerven.