VIABILITY FOR SDES IN INFINITE DIMENSIONS

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The purpose is to give conditions for the approximate viability (ε-viability) of a closed convex set K with respect to a controlled stochastic evolution equation in a real Hilbert space H. After a short introduction on finite dimensional results we come to infinite dimensional case. We allow the noise to be a cylindrical Wiener process and admit an unbounded linear operator in the state equation. We show that, if K is ε-viable, then the square of the distance from K: $d_K^2(x) := \inf_{y\in K} |x-y|^2$ is a viscosity supersolution of a suitable class of fully nonlinear Hamilton-Jacobi-Bellman equations in H. We use the definition of viscosity supersolutions for 'unbounded' elliptic equations in infinite variables that has been recently introduced by Swiech 94. Finally, we show how the general condition yields specific and natural conditions when K is a ball or a finite dimensional linear subspace of H and we prove that in these cases, under more restrictive assumptions, the necessary condition is also sufficient. Finally we give example of application to stochastic heat equation driven by white noise in dimension 1.