Some second-order necessary conditions for differential-difference inclusion problems

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May 25, 2006

Consider the following problem

$$\text{minimize} \quad g(x(T))$$

subject to

$$\begin{aligned}
&x'(t) \in F(t, x(t), x(t-\Delta)) \quad a.e.\,([t_0, T]),\\
&x(t) = c(t) \quad t \in [t_0 - \Delta, t_0),\\
&x(t_0) \in X_0, \quad x(T) \in X_1,
\end{aligned}$$

where $X_0, X_1 \subset R^n$ are given subsets, $F(.,.,.) : R \times R^n \times R^n \to \mathcal{P}(R^n)$ is a given set-valued map, $g(.) : R^n \to R$ is a given function, $\Delta \in (0, T - t_0)$ and $c(.) : [t_0 - \Delta, t_0) \to R^n$ is a given essentially bounded function.

We propose an approach concerning second-order optimality conditions for this problem. The main idea is to reduce the (infinite-dimensional) optimal control problem to the finite-dimensional problem of minimizing the terminal payoff on the intersection of the (known) target set with the (unknown) reachable set and to use a general result from nonsmooth analysis.

In order to apply the general abstract optimality conditions we must check a certain constraint qualification, so we are naturally led to study the intermediate tangent cone and Clarke tangent cone to the reachable set at the optimal endpoint.