Portfolio optimisation with economic factors and transaction costs

In this talk I would like to present two results concerning existence of optimal strategies in portfolio problems with transaction costs and economic factors. The costs are bounded away from zero (e.g. constant plus proportional costs), which requires the use of impulsive control methodology. First problem treats a continuous-time model with the functional which is a sum of the reward of the consumption and the integral-type term depending on the portofolio and the state of the market. Under quite general transaction costs struncture existence of an optimal strategy that comes from a solution to appropriate Bellman equation is shown. Various examples are presented. Second problem deals with a discrete-time model, where a so called long-run average cost functional is considered, i.e. it measures average yield of the portfolio per unit time. This type of functionals presents a considerable difficulty. Existence of optimal strategy is obtained. Under additional assumptions this strategy is proved to come from a solution to appropriate Bellman equation. A characteristic trait of both results is that the methods used in the proofs are of functional character. We do not exploit the differential approach (quasi-variational inequalities). This allows us to obtain results in greater generality. However, when one knows that the value function solves the Bellman equation and is appropriately smooth, it is a good starting point for numerical analysis. This is a major motivation for the performed research.

More details are given below.

Consider a market modeled by a time homogeneous Markov process $\big(S(t), X(t)\big)_{t\geq 0}$, where $S(t) = \big(S^1(t), \dots, S^d(t)\big) \in (0,\infty)^d$ denotes prices of d assets and $X(t) \in \mathbb{R}^m$ stands for m economic factors. Economic factors expand capabilities of the model to a great extent and facilitates and improves the quality of statistical estimation of the model parameters. Assume that $\big(S(t), X(t)\big)$ satisfies the so called Feller property. Let $N^i(t)$ be the number of shares of the i-th asset in the investor's portfolio at time $t \geq 0$. The vector $N(t) = \big(N^1(t), \dots, N^d(t)\big)$ describes the portfolio contents at time t. We shall assume that $N^i(t) \in [0,\infty)$, $i = 1, \dots, d$, which means that neither short-selling nor borrowing is allowed. Let $C_k \in [0,\infty)$ be the amount of money withdrawn from the portfolio and consumed at time τ_k, $k = 1, 2, \dots$. We are interested in maximization of the discounted infinite horizon reward functional

$$\mathbb{E}\Big\{\int_0^\infty e^{-\alpha s} F\big(Y(s)\big) ds + \sum_{k=1}^\infty e^{-\alpha\tau_k} G(C_k)\Big\},$$

where $Y(s) = \big(N(s), S(s), X(s)\big)$, F, G are continuous functions. We prove that the value function is cotinuous and satisfies the Bellman equation linked to that problem. We do not require that F, G be bounded. We show that assumptions needed for validity of the result are satisfied in a large class of well-established models.

For the second mentioned problem assume that the model is in discrete time and consider the functional

$$\liminf_{T\to\infty} \frac{1}{T}\mathbb{E}\,\ln\big(N(T)\cdot S(T)\big)$$

that we want to maximise. This type of optimisation is usually treated with vanishing discount approach. In general we cannot prove that the value function is a solution to the Bellman equation related to the problem. We obtain the optimal strategy as a limit of optimal strategies for related discounted problems. Under additional assumptions we are able to show that the value function solves the Bellman equation. Moreover, the proof is adaptable to the case with incomplete observation of economic factors.

The above results are contained in two papers:

J. Palczewski, Ł. Stettner *Impulsive control of portfolios,* to appear in Applied Mathematics and Optimisation

J. Palczewski, Ł. Stettner *Optimisation of portfolio growth rate on the market with constant plus proportional transaction costs,* Working paper