Discretization of SDEs: Euler Methods and Beyond

Given $T > 0$ and functions $\mu : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma : [0,T] \times \mathbb{R}^n \to \mathbb{R}^{n \times d}$ satisfying some regularity conditions. We study the Stochastic Differential Equation (SDE)

$$\begin{cases} dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dB_t & t \in [0,T] \\ S_0 = s_0 \end{cases},$$

where B is a d-dimensional Brownian motion on $[0,T]$. SDEs of this type – and even more general ones – are used in financial mathematics as models for the time-evolution of stocks. Furthermore, the Feynman-Kac-formula implies that an important class of PDEs can be represented using the solution to SDEs. Naturally, explicit solutions to SDEs can usually not be given.

The topic of the talk is approximation methods for SDEs. We focus on weak approximation, i. e. we consider approximating processes $S^{(m)}$ such that

$$E\big(f(S_T^{(m)})\big) \to E\big(f(S_T)\big), \quad m \to \infty,$$

for all functions $f : \mathbb{R}^n \to \mathbb{R}$ regular enough. We start with discussing construction of the approximating processes using a straightforward generalisation of the classical Euler method for ODEs, continue with higher order methods and also discuss some generalisations to other situations.