An Implementation of the LIBOR Market Model for Pricing Exotic Constant Maturity Swaps

Stefan Gerhold

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Overview

- ÖBFA (Austrian Federal Financing Agency) has a portfolio of exotic swaps
- First goal: Implement a pricing engine
- Needed for collateral management
- Less exact pricing required than for trading

The Libor Market Model (Brace et al. 1997)

- Based on observable market rates
- Models a discretization of the yield curve
- Recovers Black's formula for caplets
- Transparent modelling of correlation of different points on the yield curve
- Industry standard for pricing exotic interest rate derivatives

Forward Rates in the LMM

- Fix a tenor structure $T_0 < \cdots < T_M$
- Let $F_k(t) = F(t; T_{k-1}, T_k)$ be the simply compounded interest rate prevailing at time t for borrowing from T_{k-1} to T_k
- For each modelled rate, reset T_{k-1} and maturity T_k are fixed, while time t increases
- Example: If $T_k - T_{k-1} = 3$ months, then $F_k(T_{k-1})$ is the 3-months-EURIBOR at T_{k-1}. At smaller time t, it is a forward 3-months-EURIBOR.

Forward Rates and Forward Measures

- The LMM models a vector of spanning forward rates
- $F_k(t) = \frac{1}{T_k - T_{k-1}} \left(\frac{B(t, T_{k-1})}{B(t, T_k)} - 1 \right)$, with $B(\cdot, \cdot) =$ zero coupon bond
- Hence $F_k(t)$ is a martingale under the T_k-forward measure
- $F_k(t)$ is *not* a martingale under other T-forward measures
- Drift of $F_k(t)$ under T_j is a rational function of the other rates

An Example

- A structured note with 27 semi-annual coupons
- Coupon at time T_k is
$$\begin{cases} \text{EUR6M} + \text{const} & \text{if EUR10Y} > \text{EUR2Y} \\ 0 & \text{otherwise} \end{cases}$$
- EUR6M= 6 months EURIBOR, EUR2Y = 2 years Euro swap rate
- Coupon at T_k depends on $F_k, \ldots, F_{k+19}$ at T_{k-1}
- Additionally, there is a floor on the whole structure

Volatility Calibration

- ÖBFA provides cap volatilities for maturities 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, 10Y
- Volatilities for intermediate maturities are interpolated from market data
- We assume that caps on EUR3M have the same vol as caps on EUR6M
- Bootstrap caplet volatilities

Volatility Calibration

- Volatility of $F_k(t)$ is (following Rebonato)

$$\sigma_k(t) = \phi_k(a + b(T_{k-1} - t))\mathrm{e}^{-c(T_{k-1}-t)} + d$$

- Parameters a, b, c, d are found by minimization with Mathematica
- Fairly good fit to swaption volatility surface

Correlation Calibration

- Parametric structure $\rho_{ij} = \ell + (1 - \ell)\mathrm{e}^{-\theta|i-j|}$
- Best fit to historical data for $\ell = 0.49$, $\theta = 0.13$
- Positive correlations
- Decrease when moving away from main diagonal
- Could also be calibrated to swaption volatility matrix

Monte Carlo Simulation

- Use dynamics of F under terminal measure

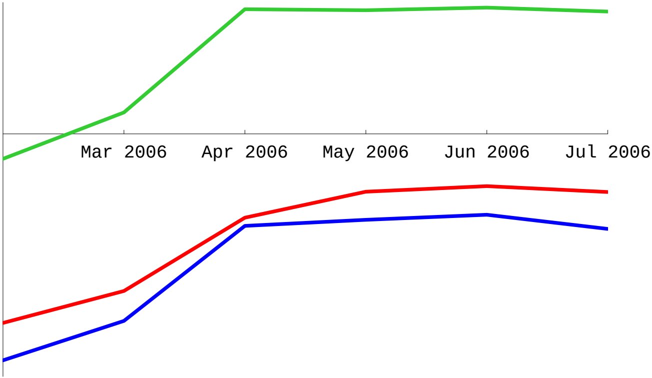
$$\frac{\mathrm{d}F_k(t)}{F_k(t)} = \mu(F_{k+1}(t), \ldots, F_M(t))\,\mathrm{d}t + \sigma_k(t)\,\mathrm{d}W_k(t)$$

- W Brownian motion with instantaneous correlation matrix $\boldsymbol{\rho}$
- $W(t + \Delta t) - W(t) \sim \sqrt{\Delta t}\,\mathcal{N}(0, \boldsymbol{\rho})$
- Parameters: Number of time discretization points, number of Monte Carlo trials, number of factors

Parameters of the Monte Carlo Simulation

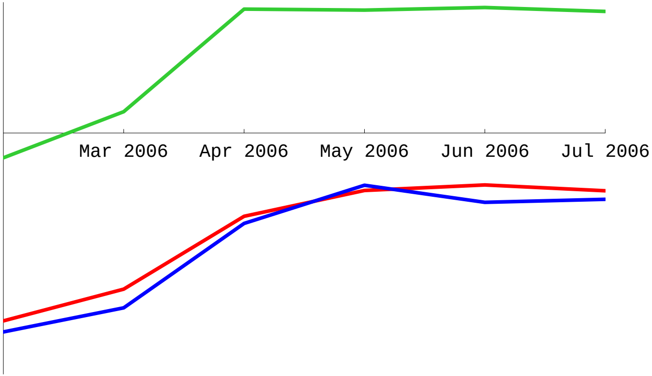
- Simulation was done with the full number of factors (46)
- 3 discretization steps per interest period
- 5000 Monte Carlo trials
- Computation time $\approx$ 40 min

Comparison of the Valuations



Old ÖBFA valuation (green), new ÖBFA valuation (blue), UBS valuation (red)

Comparison of the Valuations with higher long term correlation



Now $\ell = 0.6$. Old ÖBFA valuation (green), new ÖBFA valuation (blue), UBS valuation (red)

Conclusion

- Previous ÖBFA evaluations had problems, in particular with path dependence
- We can reproduce the counterpart evaluation for several structured notes
- Model choices seem to be sufficient for collateral management
- Mathematica + UnRisk provide comfortable implementation
- Next step: early exercise products