An affine intensity model for large credit portfolios

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The model

Related literature

Model calibration

Simulations

Conclusions

Joint work with Beatrice Acciaio

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- We model the intensity of default of N obligors.
- We assume that the obligors belong to S different groups (industrial sectors) and to a unique general environment (market).
- The default of any obligor can be due to one of the following causes (credit events):
  1. idiosyncratic
  2. sectoral
  3. general

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The model

- The time of default τ_j of obligor j has intensity

$$\lambda_j = X_j + u_j Y_{g(j)} + v_j Z, \quad j = 1, \ldots, N,$$

where $\{X_1, \ldots, X_N, Y_1, \ldots, Y_S, Z\}$ are independent affine processes. $X_1, \ldots X_N$ are the idiosyncratic factors, $Y_1, \ldots, Y_S$ are the sectoral factors Z is the common factor.

- The factors are affine jump-diffusions,

$$dX(t) = k(\theta - X(t))dt + \sigma\sqrt{X(t)}dW(t) + dJ(t),$$

where W is Brownian motion, J a pure-jump process with jump sizes exponential with mean μ, and jump times independent Poisson process with arrival rate ℓ.

- Note: λ_j in general is not affine.

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Related literature

- We generalize a model by Duffie and Gârleanu (2001) for the valuation of CDO's. DG restrict λ_j's to be affine, consider only one common factor, imposing to all the obligors the same sensibility to it.
- DG illustrate the effect of correlation and prioritization on CDO's valuation
- A common critique: intensity models produce low correlations of defaults (Hull and White (2001))
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Survival probabilities

- The survival probability

$$\begin{aligned}\mathbb{P}(\tau_j > t) &= \mathbb{E}_0 e^{-\int_0^t \lambda_j(u)du} \\ &= \mathbb{E}_0 e^{-\int_0^t X_j(u)du} e^{-u_j \int_0^t Y_{g(j)}(u)du} e^{-v_j \int_0^t Z(u)du} \\ &= \mathbb{E}_0 e^{-\int_0^t X_j(u)du} \mathbb{E}_0 e^{-u_j \int_0^t Y_{g(j)}(u)du} \mathbb{E}_0 e^{-v_j \int_0^t Z(u)du}\end{aligned}$$

... because the processes are affine:

$$f(X, t, m) := \mathbb{E}_0 e^{-\int_0^t mX(u)du} = \exp(C(X, t, m)),$$

where

$$C(X, t, m) = \alpha_\psi^m(t) + \beta_\psi^m(t) X(0).$$

The functions $\alpha_\psi^m(t), \beta_\psi^m(t)$ can be efficiently computed.

Factorization of survival probabilities

- The survival probabilities of the i-th obligor can be factorized as

$$s_i^t := f(X_i, t, 1) f(Y_{g(i)}, t, u_i) f(Z, t, v_i),$$

  The formula shows the dependence of the default probability of a single obligor on the occurrences of idiosyncratic, sectoral, general "credit events".
- Let $e(M, t)$ be the r.v. indicating the occurrence of a general credit event before time t, then

$$\mathbb{P}(e(M, t)) = 1 - f(Z, t, 1)$$

  and

$$\mathbb{P}(\tau^i < t, e(M, t)) = 1 - f(Z, t, v_i))$$

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Data needed for calibration

- For each obligor i, the survival probabilities

$$s_i^t := \mathbb{P}(\tau_i > t), \quad i = 1, \ldots, N, \quad t \in \mathcal{T}.$$

- The probabilities of general and sectoral credit events

$$p^M(t) := \mathbb{P}(e(M, t))$$

$$p^i(t) := \mathbb{P}(e(G(i), t)), \quad i = 1, \ldots, S$$

- For each obligor i, the probability of default conditioned on a general or sectoral credit event

$$p_{i|G}(t) := \mathbb{P}(\tau^i < t | e(G(i), t))$$

$$p_{i|M}(t) := \mathbb{P}(\tau^i < t | e(M, t))$$

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Calibration procedure

- Estimate the parameters sets $\hat{\psi}_c$ and $\{\hat{\psi}_{g_i},\ i = 1, ..., S\}$ that better fit the input data $\{p^M(t),\ t \in \mathcal{T}\}$ and $\{p^i(t),\ i = 1, ..., S, t \in \mathcal{T}\}$
- From $p_{i|G}(t)$ and $p^i(t)$, extract $\mathbb{P}(\tau^i < t, e(G(i), t))$ and compute u_i from

$$u_i := \text{argmin}_u \|\mathbb{P}(\tau^i < t, e(G(i), t)) - (1 - f(Y_{g_i}, t, u))\|$$

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- Fit the obligors' survival probabilities s_i^t by calibrating the parameters of the idiosyncratic factor X_i

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An example of calibration

- An exercise with default probabilities taken from Moody's transition matrix 1980-1999.
- We consider a portfolio of 90 obligors, belonging to 3 groups.
- Within each group, obligors 1-10 are Ba, 11-20 are B, 21-30 are Caa ▸ Plot
- General credit events are rare (Aa) ▸ Plot
- Groups credit events are (Baa,Ba,B) ▸ Plot

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- The dependence is only considered at time $t = 1$
- We assume three levels of dependence $L = 0.25$; $M = 0.5$; $H = 0.75$, that is

$$p_{i|G}(t) = L, M, H$$

- Within each group, obligors 1-10 have dependence (with their group and with the general factor) L, 11-20 M, 21-30 H.
  ▸ Plot
- We can compute the correlation of defaults before time t

$$\rho_{i,j} = \text{corr}\left(\mathbf{1}_{\tau^i<t}, \mathbf{1}_{\tau^j<t}\right)$$

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Simulations

The standard way to simulate default scenarios $\left\{\tau^{(n)}, I^{(n)}\right\}_{n=1}^{N_d}$ is

- From $\lambda_j(t_0), j = 1, \ldots, N$ compute the total intensity $\Lambda(t_0) = \sum_{i=1}^{N} \lambda_j(t_0)$
- Simulate the occurrence of one default in the interval Δt with probability $\Lambda(t_0)\Delta t$
- In case of default extract the identity of defaulter with probabilities $p_j = \lambda_j(t_0)/\Lambda(t_0)$
- Compute $\lambda_j(t_0 + \Delta t), j = 1, \ldots, N$ by discretization

Simulations

- The standard way to simulate is based on the discretization of the continuous process. It converges as $\Delta t \rightarrow 0$
- The smaller Δt the greater the computational effort
- Another problem is that the discretization of affine-square root processes often produces negative values.

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Exact simulations

Exact simulation for default scenarios $\left\{\tau^{(n)}, I^{(n)}\right\}_{n=1}^{N_d}$

- I set $\tau^{(0)} := 0$, $C_0 := \{1, ..., N\}$ and $n := 1$;
- II simulate $\tau^{(n)}$, IF $\tau^{(n)} > T$ or $n = N$ STOP;
- III extract $I^{(n)}$;
- IV re-start the intensities λ_j of obligors $j \in C_n := C_{n-1} \setminus \{I^{(n)}\}$;
- V set $n = n + 1$, GO TO (II).

A comparison of the two methods

- We simulated default scenarios from the previous calibration exercise
- 10 simulations for defaults before time $T = 7$
- Consider the discretization method. Let N be the number of intervals in a year.
- Compute the mean frequency of negative values as a function of N [Plot]
- Compute the CPU times for the 10 simulations and compare to the exact method [Link]

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Conclusions

- A model for large portfolios subject to credit risk (e.g. CDO)
- The affine setting leads to closed formulas for many important quantities
- Flexible enough for calibration
- Can produce high correlations between defaulters
- Efficient production of default scenarios through exact simulation

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II: Simulate the n^{th} default-time $\tau^{(n)}$

- Invert the total survival probability

$$\begin{aligned} SP_n(t) &:= \mathbb{P}(s_n > t | \mathcal{F}_{\tau^{(n-1)}}) \\ &= \mathbb{E}_{\tau^{(n-1)}}\left[e^{-\int_{\tau^{(n-1)}}^{\tau^{(n-1)}+t} \Lambda^{n-1}(u)du} \right] \end{aligned}$$

where

$$\Lambda^{n-1}(t) = \sum_{j \in C_{n-1}} \lambda_j(t) = \sum_{j \in C_{n-1}} X_j(t) + u_j Y_{g(j)}(t) + v_j Z(t)$$

- Easy to be computed and inverted (numerically).

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III: extract the n^{th} defaulter

- The probability, conditioned to $\tau^{(n)}$, that $I^{(n)} = j$ is

$$\mathbb{P}\left(I^{(n)} = j | \mathcal{F}_{\tau^{(n-1)}} \vee \tau^{(n)}\right) = \frac{\mathbb{E}_{\tau^{(n-1)}}\left[\lambda_j(\tau^{(n)}) e^{-\int_{\tau^{(n-1)}}^{\tau^{(n)}} \Lambda^{n-1}(s)ds}\right]}{\mathbb{E}_{\tau^{(n-1)}}\left[\Lambda^{n-1}(\tau^{(n)}) e^{-\int_{\tau^{(n-1)}}^{\tau^{(n)}} \Lambda^{n-1}(s)ds}\right]}, \quad \forall$$

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IV: re-start the intensities

- Suppose $\tau^{(1)} = t$ and $I^{(1)} = k$, then

$$\mathbb{E}[\lambda_j(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k] = \mathbb{E}[X_j(\tau^{(1)}) + u_j Y_{g(j)}(\tau^{(1)}) + v_j Z(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k].$$

- $$\mathbb{E}[X_j(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k] = \frac{\partial}{\partial r}\Big|_{r=0} H^j(r; t, k), \; j = 1, ..., N$$

- $$H^j(r; t, k) := \mathbb{E}[e^{rX_j(\tau^{(1)})}|\tau^{(1)} = t, I^{(1)} = k] = \frac{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)e^{rX_j(t)}]}{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)]}.$$

- Easily computed in affine setting Back

IV: re-start the intensities

- Suppose $\tau^{(1)} = t$ and $I^{(1)} = k$, then

$$\mathbb{E}[\lambda_j(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k] = \mathbb{E}[X_j(\tau^{(1)}) + u_j Y_{g(j)}(\tau^{(1)}) + v_j Z(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k].$$

-

$$\mathbb{E}[X_j(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k] = \frac{\partial}{\partial r}\Big|_{r=0} H^j(r; t, k), \; j = 1, ..., N$$

-

$$\begin{aligned} H^j(r; t, k) &:= \mathbb{E}[e^{rX_j(\tau^{(1)})}|\tau^{(1)} = t, I^{(1)} = k] \\ &= \frac{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)e^{rX_j(t)}]}{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)]}. \end{aligned}$$

- Easily computed in affine setting Back

IV: re-start the intensities

- Suppose $\tau^{(1)} = t$ and $I^{(1)} = k$, then

$$\mathbb{E}[\lambda_j(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k] = \mathbb{E}[X_j(\tau^{(1)}) + u_j Y_{g(j)}(\tau^{(1)}) + v_j Z(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k].$$

- $$\mathbb{E}[X_j(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k] = \frac{\partial}{\partial r}\Big|_{r=0} H^j(r; t, k), \; j = 1, ..., N$$

- $$\begin{aligned} H^j(r; t, k) &:= \mathbb{E}[e^{rX_j(\tau^{(1)})}|\tau^{(1)} = t, I^{(1)} = k] \\ &= \frac{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)e^{rX_j(t)}]}{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)]}. \end{aligned}$$

- Easily computed in affine setting Back

IV: re-start the intensities

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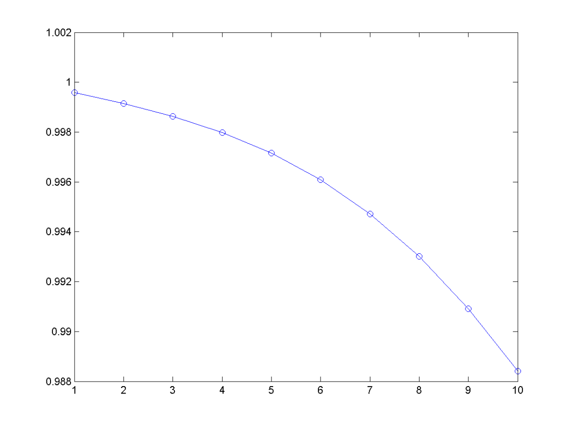
- $$\mathbb{E}[X_j(\tau^{(1)})|\tau^{(1)} = t, I^{(1)} = k] \quad = \quad \frac{\partial}{\partial r}\Big|_{r=0} H^j(r; t, k), \; j = 1, ..., N$$

- $$\begin{aligned} H^j(r; t, k) \quad &:= \quad \mathbb{E}[e^{rX_j(\tau^{(1)})}|\tau^{(1)} = t, I^{(1)} = k] \\ &= \quad \frac{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)e^{rX_j(t)}]}{\mathbb{E}[e^{-\int_0^t \Lambda(s)ds}\lambda_k(t)]}. \end{aligned}$$

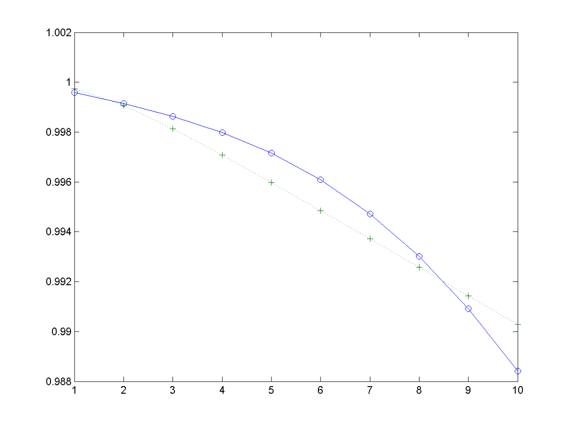
- Easily computed in affine setting Back

Input data: General factor (Aa)



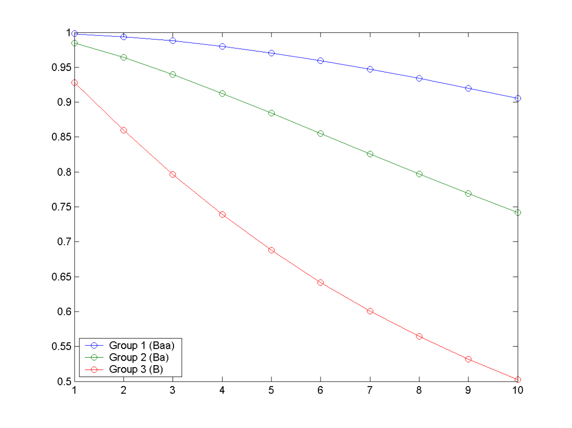
Text Fitting

Calibrated data: General factor (Aa)



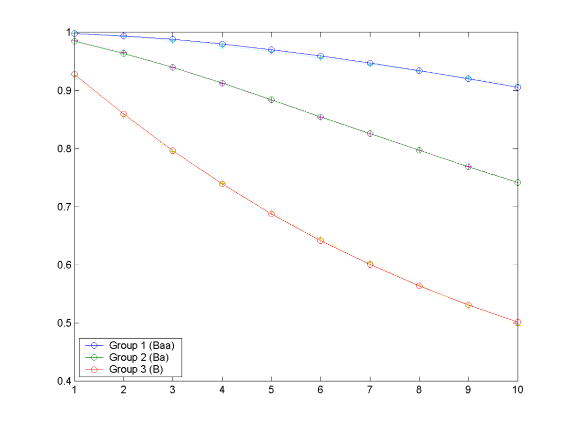
Text

Input data: Groups



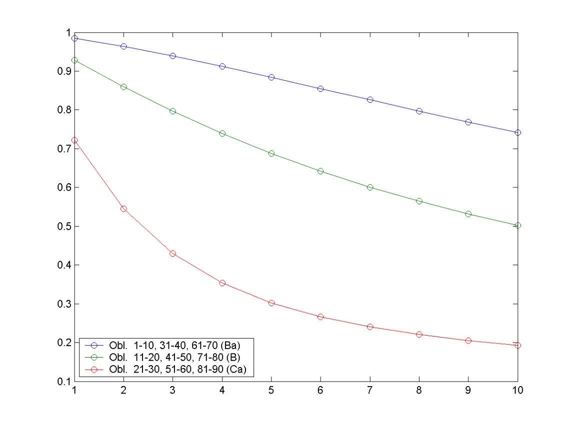
Text Fitting

Calibrated data: Groups



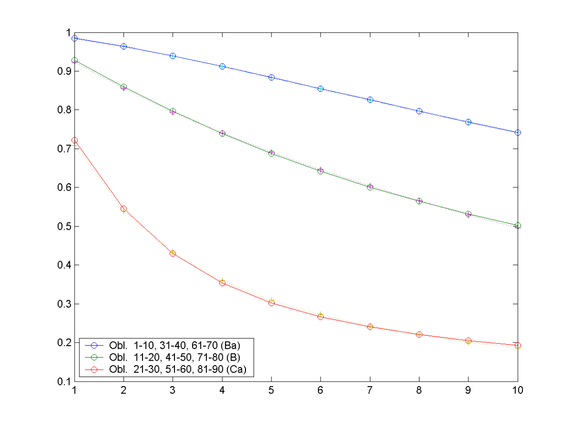
▸ Text

Input data: obligors



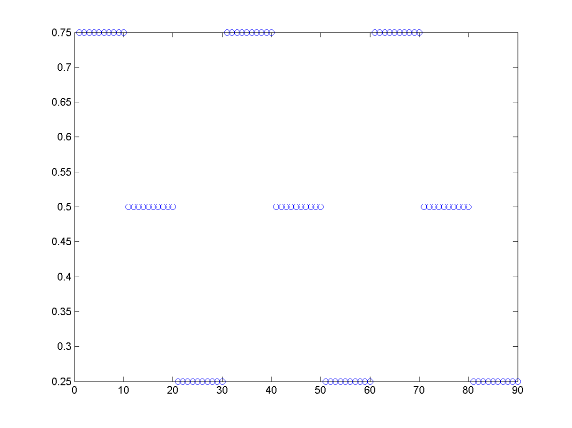
Text Fitting

Calibrated data: obligors



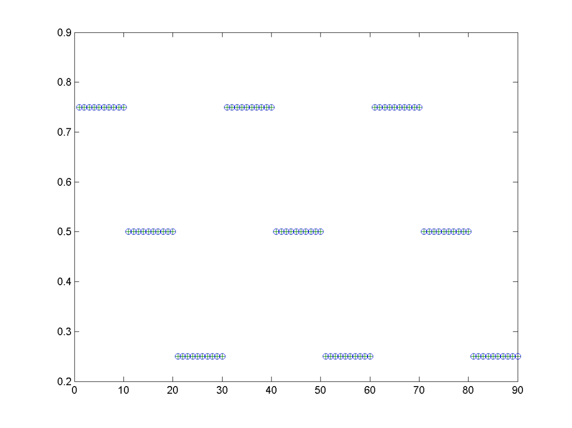
Text

Input data: conditional probabilities (general and sectoral)



Text Fitting

Cal. data: conditional probabilities (general and sectoral)



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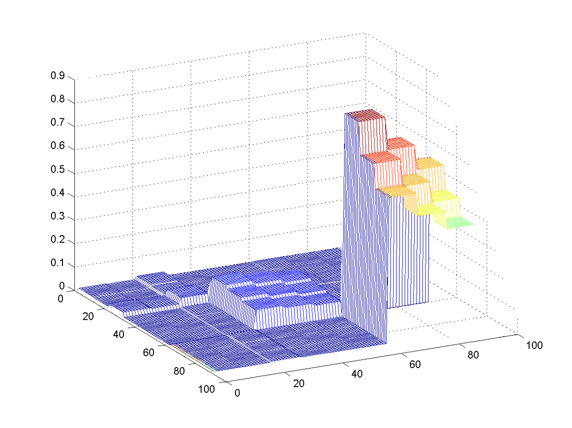
Correlation of defaults in the first year



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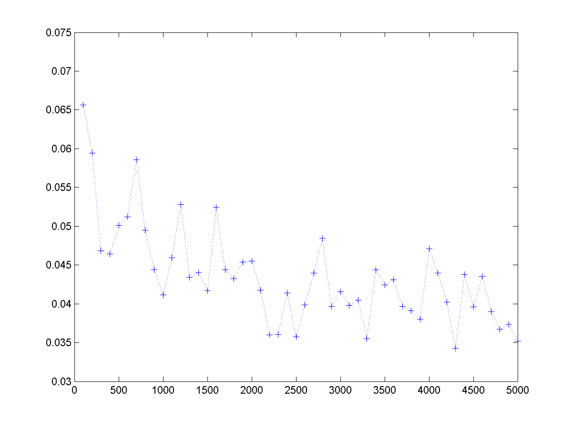
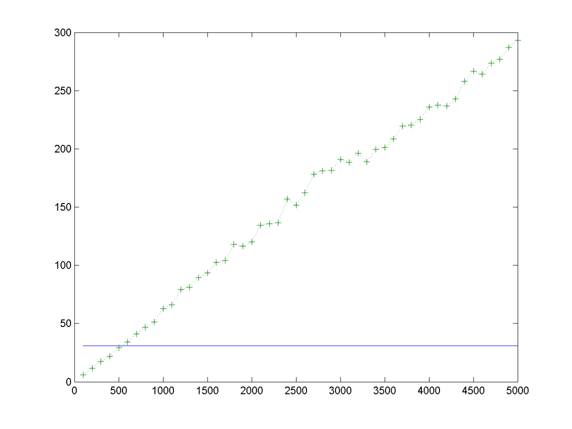
Frequency of Negative Values



Text

CPU times



Text