Risk Management and Saving: Income Effects and Background Risk

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Abstract

We study the interplay of intertemporal risk management and saving decisions. We define risk management broadly by allowing the activity to influence the severity of loss, the probability of loss or both simultaneously. Due to the similar cost-benefit structure of risk management and saving decisions a substitution effect arises whose implications we analyze for changes in income and background risk. Typically, the direct effects for risk management and saving move in the same direction but because of substitution net effects become a priori ambiguous. We resolve this ambiguity by deriving necessary and sufficient conditions. Our paper cautions against the use of single-instrument models as spurious results will emerge.

Keywords: risk management · saving · income effects · background risk · substitution.

JEL-Classification: D81 · D90 · D91 · E21

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1 Introduction

In the face of risk individuals will consider risk management to mitigate the occurrence and severity of untoward events. A classical distinction has been made between self-insurance or loss reduction, which reduces the severity of loss, and self-protection or loss prevention, which reduces the probability of loss (Ehrlich and Becker, 1972). Whereas the first activity is a substitute for insurance, the latter can substitute or complement insurance coverage.[1] Furthermore, more risk-averse agents in the sense of Pratt (1964) invest more in self-insurance but not necessarily in self-protection (Dionne and Eeckhoudt, 1985; Briys and Schlesinger, 1990; Jullien et al., 1999). Also wealth or income effects are different for the two forms of risk reduction (Sweeney and Beard, 1992; Lee, 2005). As a result, self-insurance and self-protection decisions have often been analyzed separately.

As pointed out by Lee (1998), risk management may affect both the probability and severity of loss simultaneously.[2] We use this more general notion and put it into an intertemporal context. To mitigate (anticipated) future consumption risks, agents invest today in a costly activity that increases expected future consumption utility. This intertemporal view on risk management decisions has gained momentum in recent years.[3] However, as soon as we apply it, alternative means of responding to future consumption risk are at hand.

Saving shifts wealth from today into the future and makes a given consumption risk easier to bear for a risk-averse individual.[4] As such it appears as a natural substitute for any risk management activity. Indeed, both instruments trade off current consumption against an increase in expected future consumption. We show that this substitution effect holds not only for self-protection (Menegatti and Rebessi, 2011) but also for the generic class of risk management activities considered in this paper. It is a universal phenomenon whose implications for income

[1] This also holds outside the realm of expected utility as established by Courbage (2001) with Yaari's (1987) dual theory of choice.

[2] Engaging a good lawyer reduces the probability of conviction and the punishment for crime. Proper vehicle maintenance and cautious driving reduce the likelihood of an accident and the magnitude of loss in case an accident occurs. Regular medical checkups decrease the probability and severity of future illnesses.

[3] Menegatti (2009) analyzes self-protection and prudence in a two-period model and obtains opposite results as Eeckhoudt and Gollier (2005) who use a monoperiodic model. Menegatti and Rebessi (2011) combine self-protection and saving and also Hofmann and Peter (2014) document the importance of saving for the comparative effect of individual preferences on risk management decisions.

[4] Drèze and Modigliani (1972) study consumption decisions when agents are exposed to risk and the comparative effects of their intertemporal preferences. Kimball (1990) points out that prudence, i.e., a positive third derivative of utility, is the necessary and sufficient condition for future income uncertainty to induce precautionary wealth accumulation. Bommier et al. (2012) derive general results on the impact of risk aversion on saving motives thereby clarifying the link between prudence and risk aversion.

effects and background risk are pointed out in this paper. As a consequence, studying risk management or saving *in isolation* will necessarily render spurious predictions.

To give an example, many countries have or had tax subsidized saving programs in place (Engelhardt, 1996; Bovenberg, 1989). An increase in the tax subsidy makes saving more attractive. Still, empirical results for Individual Retirement accounts were mixed (Venti and Wise, 1990; Gale and Scholz, 1994). Our results suggest that a negative substitution effect arising from risk management activities may outweigh positive effects on saving so that overall savings decrease following an exogenous shock. Furthermore, the (unobservable) use of risk management activities and heterogeneity therein represent a potential channel for explaining heterogeneity in saving responses. Conversely, incentivizing safety investments can also lead to less risk management rather than more if the substitution effect from saving is dominant. In short, as soon as risk management and saving are considered simultaneously to optimize expected intertemporal consumption utility, the comparative statics are governed by the substitution effect which implies the ambiguity of the majority of exogenous changes. Resolving this ambiguity requires knowledge about the structure of the underlying decision problem implying that there are no generic conditions that can be universally applied.

In the next section we develop a simple model of risk management and saving. We discuss comparative statics which are absolutely straightforward and unambiguous for risk management but become ambiguous in the presence of saving. Section 3 introduces background risk and shows how the optimal risk management and saving portfolio reacts to changes in the size of a background risk. In section 4, we extend the result that risk management and saving are substitutes to various contexts. We show that it is a universal phenomenon which is present in a huge variety of contexts. The last section concludes.

2 A Simple Model

2.1 Notations

We consider a decision-maker (DM) who lives for two periods, t_1 and t_2, now and then. She receives income w_i in t_i where we assume for the sake of simplicity that first-period income is risk-free. However, second-period income is subject to a random loss of l that occurs with probability p. Risk and time preferences are characterized by the first-period felicity function U, the second-period felicity function V and a rate of pure preference for the present of β. U

and V are assumed to be increasing and concave which reflects non-satiation and risk aversion of the DM. Consequently, expected utility of intertemporal consumption is given by

$$EU = U(w_1) + \beta\left[pV(w_2 - l) + (1-p)V(w_2)\right].$$

2.2 Risk Management

To reduce the second-period consumption risk individuals can pursue a costly risk management activity. We denote by e the intensity of this activity ("effort") and by $c(e)$ the cost associated with it. Naturally, $c(0) = 0$ and $c' > 0$, and we also assume that $c'' \geq 0$. Furthermore, we make the assumption that effort may affect the probability of loss ($p' \leq 0$) and/or the severity of loss ($l' \leq 0$) with at least one inequality strict. This modeling is very flexible as it comprises self-protection and self-insurance decisions (Ehrlich and Becker, 1972) as well as combinations of the two, so called self-insurance-cum-protection decisions (Lee, 1998).[5]

Hence, the individual's objective function is given by

$$\max_e EU(e) = U(w_1 - c(e)) + \beta[p(e)V(w_2 - l(e)) + (1 - p(e))V(w_2)]. \tag{1}$$

Throughout the entire paper we consider interior solutions that can be characterized via the first-order conditions. For (1) we obtain

$$EU_e = -c'U_1' + \beta p'[V_{2L} - V_{2N}] - \beta p l' V_{2L}' = 0, \tag{2}$$

where we denote $U_1 := U(w_1 - c(e))$, $V_{2L} := V(w_2 - l(e))$, and $V_{2N} := V(w_2)$ to compress notation.[6] Note that the marginal cost from investing in risk management is given by the first term as measured in utility units, i.e., marginal expenditures on risk management times marginal utility of first-period consumption. The marginal benefit consists of the second term, which captures the effect of a reduction of the probability of loss on expected utility, and the third term, which measures the impact of a less severe loss on expected utility.

[5] As a special case of self-insurance also market insurance decisions are covered in our model, i.e. a coinsurance contract for which the premium and the retained loss are linear functions of coverage.

[6] Under standard assumptions on the cost ($c' > 0,\ c'' \geq 0$) and the risk management technology ($p' \leq 0,\ p'' \geq 0,\ l' \leq 0,\ l'' \geq 0$), the objective function is concave in the intensity of risk management,

$$-c''U_1' + (c')^2U_1'' + \beta p''(V_{2L} - V_{2N}) - 2\beta p' l' V_{2L}' - \beta p l'' V_{2L}' + \beta p (l')^2 V_{2L}'' < 0.$$

Note that the dependence of p on e does not create non-concavities in expected utility here as opposed to single-period models (Shavell, 1979).

Comparative statics are straightforward and are summarized in the following proposition.[7]

Proposition 1. *a) Higher first-period income is associated with more risk management.*

b) Higher second-period income implies a decrease in risk management.

c) Less impatience leads to an increase in risk management.

Proof. All proofs can be found in the appendix. □

The intuition is the following. If there is more income at the disposal of the DM in the first period, expenditures on risk management will hurt less in terms of forgone consumption, i.e., the marginal cost is lower. The marginal benefit, however, remains unchanged implying a positive net effect. An increase in second-period income lowers the marginal benefit from risk management. Diminishing marginal utility leads the utility difference between no-loss state and loss state to shrink as income increases, therefore the first component of the marginal benefit is decreased. The same is true for the second component of marginal benefit, i.e., more income makes a loss of the same size easier to bear. Both effects combined induce the intensity of risk management to decrease, as the marginal cost are unaffected by a change in second-period income. Weaker impatience increases the marginal benefit of both the reduced probability and severity of loss. The marginal cost is unaffected, as expenditures on risk management precede its effect. Consequently, the net effect on risk management is positive.

2.3 Risk Management and Saving

As outlined in the introduction, another way to increase expected utility of intertemporal consumption is the accumulation of savings. Let s be the DM's savings in the first period[8] with $r \geq 0$ being the interest rate which we assume to be deterministic for simplicity. The individual's objective function changes to

$$\max_{e,s} EU = U(w_1 - c(e) - s) + \beta \left[p(e)V(w_2 - l(e) + (1+r)s) + (1-p(e))V(w_2 + (1+r)s) \right].$$

[7] Besides income or wealth effects researchers have investigated the effect of risk preferences on risk management. In the last 10 years, the link between prudence or downside risk aversion and self-protection has received considerable attention (Eeckhoudt and Gollier, 2005; Chiu, 2005; Menegatti, 2009; Dionne and Li, 2011; Peter, 2013).

[8] If s is negative, this indicates borrowing against future consumption to finance current consumption.

The first-order conditions read as[9]

$$EU_e = -c'U_1' + \beta p' \left[V_{2L} - V_{2N}\right] - \beta p l' V_{2L}' = 0, \quad (3)$$

$$EU_s = -U_1' + \beta(1+r)\left[pV_{2L}' + (1-p)V_{2N}'\right] = 0. \quad (4)$$

The first condition solves the trade-off between marginal cost of investing in risk management and marginal benefit from such investment. The second condition is the usual consumption smoothing condition over the life-cycle, i.e., expected marginal utility from consumption must be equal over both periods.

If risk management and saving are considered simultaneously, the comparative statics results from Proposition (1) are ambiguous due to a substitution effect. This effect is summarized in the following remark.

Remark 1. *Risk management and saving are substitutes.*

This generalizes the finding by Menegatti and Rebessi (2011) that intertemporal self-protection and saving are substitutes and the finding by Hofmann and Peter (2014) that intertemporal self-insurance and saving are substitutes. The reason is that with higher savings there is less first-period income which increases the marginal cost of the risk management activity (see Proposition 1a). Also, with higher savings there is more second-period income which decreases the marginal benefit of risk management (see Proposition 1b). It pays less to reduce the probability or the severity of loss when the DM is richer due to diminishing marginal utility. Consequently, the net effect is negative so that an exogenous increase in saving reduces the intensity of risk management and vice versa.

To explain how this renders the comparative statics ambiguous, consider how an increase in first-period income affects the optimal level of saving. First, the marginal cost of the risk management activity is lower due to the fact that marginal utility is diminishing. The marginal benefit of risk management is unaffected when current income is increased. Therefore, we would expect an increase in the intensity of risk management (see Proposition 1a). Due to the substitution effect this would induce less savings. However, with more first-period income also saving is more attractive, as the marginal cost are lower, whereas the marginal benefit is unaffected. This would lead to an increase in saving. Hence the direct effect and the indirect

[9] We assume that the second-order conditions are satisfied. Note that $EU_{ee} < 0$ as in Footnote 6 and that $EU_{ss} = U_1'' + \beta(1+r)^2\left[pU_{2L}'' + (1-p)U_{2N}''\right] < 0$ due to risk aversion. Hence, we assume for a maximum that the determinant of the Hessian of EU be positive, $D = EU_{ee}EU_{ss} - EU_{se}^2 > 0$.

or substitution effect on saving point in opposite directions so that the net effect is ambiguous. The same holds for the remaining exogenous parameters and the risk management decision.

Consequently, it depends on the comparative efficiency of risk management and saving whether the direct or indirect effects prevail as a reaction to changes in exogenous parameters. We denote by MB^i (MC^i) the marginal benefit (marginal cost) for the risk management activity ($i = e$) and for saving ($i = s$), respectively.[10] With this notation, we obtain the following conditions

$$\begin{aligned} MB_s^e &> c' MB_s^s, && (5) \\ c' \left(MB_e^s - MC_e^s \right) &> MB_e^e - MC_e^e, && (6) \\ \frac{1}{MB_s^s} \left(MB_e^s - MC_e^s \right) &> \frac{1}{MB_s^e} \left(MB_e^e - MC_e^e \right). && (7) \end{aligned}$$

With higher second-period income it pays less to invest in risk management and to accumulate savings due to diminishing marginal utility. The intuition behind condition (5) is that this effect is weaker for risk management than for saving so that risk management is more efficient from this perspective. Condition (6) makes a different comparison. The marginal benefit of saving decreases with more risk management whereas its marginal cost increase. This is due to the substitution effect (see Remark 1). Furthermore, also the marginal benefit of risk management decreases with more risk management whereas its marginal cost increase. This is due to the second-order condition for risk management (see Footnote 6). Condition (6) states that the net effect of an increase in risk management on saving is smaller than that on risk management itself so that saving is comparatively more efficient. The last condition is the most complex one. Again, it compares how the marginal benefit and the marginal cost of risk management and saving are affected by marginal variations in the risk management decision, additionally controlling for the impact of saving and the marginal benefit of each action. The net effect on saving of the changes in consideration is smaller than the net effect on risk management so that saving is comparatively more efficient in this sense.

It is noteworthy that these conditions are neither necessary nor sufficient for one another.

[10] In detail, we have that

$$\begin{aligned} MB^e &= \beta p' \left[V_{2L} - V_{2N} \right] - \beta p l' V'_{2L}, \\ MB^s &= \beta (1+r) \left[p V'_{2L} + (1-p) V'_{2N} \right], \\ MC^e &= c' U'_1, \\ MC^s &= U'_1, \end{aligned}$$

which is obtained from the first-order conditions (3) and (4).

Said differently, knowing that one of the three holds it is not possible to conclude that one of the other two holds. Also, additional simplifications can be obtained by making the assumption that the cost of risk management are measured in currency, i.e. that $c(e) = e$.[11] Then, obviously $c'(e) = 1$ and $MC^s = MC^e = U_1'$. Conditions (5) to (7) are useful to sign the comparative statics results for optimal risk management and saving decisions. We summarize the results in the following proposition.

Proposition 2. *The comparative statics for the optimal risk management and saving are given as follows:*

a) Higher first-period income leads to more risk management if and only if (5) is satisfied.

b) Higher first-period income leads to higher savings if and only if (6) is satisfied.

c) Higher second-period income leads to more risk management if and only if (5) is satisfied.

d) Higher second-period income leads to higher savings if and only if (7) is satisfied.

e) Weaker impatience leads to more risk management if and only if (5) is satisfied.

f) Weaker impatience leads to higher savings if and only if (6) is satisfied.

As outlined above, in the model with two decisions there is a direct effect (see Proposition 1) and an indirect or substitution effect (see Remark 1), which point into opposite directions. For instance, the direct effect of more first-period income on risk management is positive but the substitution effect is negative due to the fact that also the direct effect on saving is positive. If risk management is the more efficient way to react to exogenous changes in the environment, there will be an increase. A priori it is not clear how to express comparative efficiency in this problem but the answers are given by conditions (5) - (7).[12]

3 The Effect of Background Risk

When mitigating risks DMs are often confronted with other sources of risk they cannot control. These other risks have been labelled "background risk" in the literature. One example is income

[11] One example would be the case where $c(e)$ denotes the premium for insurance coverage.

[12] Our results generalize findings for joint self-protection and saving decisions (Menegatti and Rebessi, 2011) and complement early results on insurance and saving decisions (Dionne and Eeckhoudt, 1984). We consider a much broader class of risk management technologies and show the usefulness of the comparative efficiency conditions beyond the comparative statics of income by looking at background risks.

risk which is assumed to be independent of a property risk. The existence of background risks has important consequences for optimal decision making.[13]

In this section, we will analyze the impact of background risk on optimal risk management decisions. The literature that explicitly models the intertemporal character of risk mitigation is still nascent so that existing results which mainly focus on single-period considerations cannot be applied. The findings developed in one notable exception, Courbage and Rey (2012), who focus on self-protection, still hold in our more general setting. Then, we combine risk management and saving decisions and show how the conditions developed in subsection 2.3 can be utilized to sign the different effects. Results are no longer unambiguous in the presence of saving due to the substitution effect.

3.1 Risk Management and Background Risk

Background risk may be present at different points in time and at different states of the world. Returning to the individual's objective (1) for optimal risk management, the most general way to introduce background risk would be

$$\max_{e} \ \mathbb{E}U\left(w_1 - c(e) + \widetilde{\varepsilon}_1\right) + \beta\left[p(e)\mathbb{E}V\left(w_2 - l(e) + \widetilde{\varepsilon}_{2L}\right) + (1-p(e))\mathbb{E}V\left(w_2 + \widetilde{\varepsilon}_{2N}\right)\right].$$

For simplicity we assume the risks $\widetilde{\varepsilon}_1, \widetilde{\varepsilon}_{2L}$ and $\widetilde{\varepsilon}_{2N}$ to be independent of the risk of loss and to have zero mean. In the following analysis we will distinguish between different cases.

- First-period background risk (FPBR) implies that $\widetilde{\varepsilon}_{2L} = \widetilde{\varepsilon}_{2N} \equiv 0$.
- Second-period background risk (SPBR) implies that $\widetilde{\varepsilon}_1 \equiv 0$ and $\widetilde{\varepsilon}_{2L} = \widetilde{\varepsilon}_{2N}$.
- Loss-state background risk (LSBR) implies that $\widetilde{\varepsilon}_1 = \widetilde{\varepsilon}_{2N} \equiv 0$.
- No-loss-state background risk (NSBR) implies that $\widetilde{\varepsilon}_1 = \widetilde{\varepsilon}_{2L} \equiv 0$.

The concept of state-dependent background risk was introduced by Fei and Schlesinger (2008) who study how DMs adjust their insurance consumption in the presence of state-dependent

[13] Doherty and Schlesinger (1990) show that under insurer insolvency risk with partial default, it is optimal to take out more or less than full insurance despite actuarially fair pricing. Hau (1999) generalizes this result and explains the findings by introducing full insurance coverage on average, which is the coinsurance rate that eliminates all insurable risk when the uninsurable risk is evaluated at its mean. Rey (2003) considers also non-pecuniary background risks like health risks. To evaluate how background risk affects risk preferences in general, Gollier and Pratt (1996) introduce the notion of risk vulnerability. If preferences are risk vulnerable, the introduction of an unfair background risk will make individuals behave more risk-averse. Eeckhoudt et al. (1996) study the effect of changes to the exogenous background risk on risk attitude towards endogenous risks.

background risk. The motivation for this class of background risks is that for instance income uncertainty might take different forms conditional on whether a loss obtains or not. The following proposition summarizes our results.

Proposition 3. *When the DM is prudent ($U''' > 0,\ V''' > 0$),*

a) FPBR decreases the intensity of optimal risk management,

b) SPBR increases the intensity of optimal risk management,

c) LSBR increases the intensity of optimal risk management,

d) and NSBR decreases the intensity of optimal risk management.

The intuition behind our findings is straightforward. For a prudent DM, FPBR increases the marginal cost of risk management whereas the marginal benefit is unaffected. As a result, the intensity of risk management decreases such that current consumption increases whereas expected future consumption decreases. This can be interpreted as a precautionary effect because the individual shifts wealth to the period where it bears the background risk by reducing expenditures on risk management. For a SPBR two effects can be observed. First, expected marginal utility in the loss-state increases and as a consequence the marginal benefit of reducing the severity of loss increases. Second, when future income becomes risky the utility difference between high- and low-wealth states increases for prudent DMs. Hence, also the marginal benefit of reducing the probability of loss increases. The net effect is, of course, positive so that a precautionary increase in the intensity of risk management takes place.

The remaining two cases are similar. For LSBR the marginal cost of risk management are unaffected, but its marginal benefit increases. First, the loss state is deteriorated by the appearance of background risk due to the DM's risk aversion. Consequently, it pays more to avoid this state of the world. But due to prudence also the marginal value of wealth in this state is higher so that reducing the severity of loss is beneficial as it increases consumption there. As a result the intensity of risk management is higher. For a NSBR the marginal cost of risk management are unaffected again, however the marginal benefit decreases. The reason is that due to risk aversion the introduction of a background risk deteriorates the no-loss state such that the utility difference between the loss and the no-loss state shrinks. This explains the decrease in the intensity of risk management.

Our findings generalize the results in Courbage and Rey (2012) to the broader class of risk management activities considered here. The next subsection considers saving in addition.

Precautionary effects have first been studied as related to saving (Kimball, 1990) and then extended to insurance (Eeckhoudt and Kimball, 1992) or self-protection (Courbage and Rey, 2012). We are the first to address the interplay of risk management and saving in the presence of background risk.

3.2 Risk Management, Saving and Background Risk

As a next step we include saving as a means of optimizing expected intertemporal consumption utility. Kimball (1990) was the first to show that the introduction of future income risk leads to the accumulation of precautionary savings if and only if the DM is prudent.[14] As shown in Remark 1 besides the direct effects of background risk on risk management and saving decisions a substitution effect arises according to the similar cost-benefit structure of the two instruments.

In the face of background risk, the individual's objective function when both risk management and saving are available reads as

$$\max_{e,s}\left\{\begin{array}{c}\mathbb{E}U\left(w_1-c(e)-s+\widetilde{\varepsilon}_1\right)+\\ \beta\left[p(e)\mathbb{E}V\left(w_2-l(e)+(1+r)s+\widetilde{\varepsilon}_{2L}\right)+(1-p(e))\mathbb{E}V\left(w_2+(1+r)s+\widetilde{\varepsilon}_{2N}\right)\right]\end{array}\right\}.$$

Paralleling the previous subsection we investigate background risks that might occur at different points in time and different states of the world. We start with FPBR. To use standard comparative statics techniques, we assume $\widetilde{\varepsilon}_1 = k \cdot \widetilde{\varepsilon}$ and vary the size of the background risk by considering changes in the scale parameter $k \geq 0$. This sheds light on how the intensity of risk management and saving are determined in the face of background risk and where the substitution effect comes into play.

As in subsection 2.3 the comparative efficiency of risk management and saving has to be considered to determine the direction of the resulting effects when the size of a background risk changes. In the case of a FPBR, we can "recycle" condition (5) from above and need the following extension of condition (6):

$$c'\left(MB_e^s - \mathbb{E}\widetilde{MC}_s^e\right) > MB_e^e - \mathbb{E}\widetilde{MC}_e^e. \tag{8}$$

Here $\widetilde{MC}$ denotes marginal cost in the presence of background risk. Note that if condition (5)

[14] Furthermore, the intensity of prudence as measured by the coefficienty of absolute prudence $-V'''/V''$ is monotonically related to the strength of the precautionary saving motive.

holds for all income levels w_1 it follows that (8) is satisfied because taking expectations preserves the inequality. We obtain the following proposition.

Proposition 4.a. *When the DM is prudent in the first period ($U''' > 0$), increasing the size of a FPBR*

- *leads to less risk management if and only if (5) is satisfied,*
- *leads to lower savings if and only if (8) is satisfied.*

This reveals that background risk in the first period has an effect on the DM's optimal decisions that is closely related to the effect of decreasing first-period income (see Proposition 2 a) and b)). The intuition for this result can be obtained by using the notion of the precautionary premium (Kimball, 1990). It is defined as the sure wealth reduction implying the same marginal utility as a zero-mean risk and is positive for prudent agents.[15] Consequently, the effect of increasing the size of a FPBR on marginal utility is identical to a sure reduction of first-period income when the DM is prudent. As a result the marginal cost of both risk management and saving increase whereas marginal benefits remain unaffected. Hence, there is a negative direct effect on both instruments and – as a consequence – a positive substitution effect (Remark 1) from each instrument on the other. Condition (5) tells us when the direct effect dominates for risk management and condition (8) tells us when this is the case for saving.

The case of SPBR is much more involved. Unlike in the previous case, here not only marginal utility but also utility is affected. Due to the fact that the precautionary premium and the risk premium associated with a background risk need not be identical, changes in the size of an SPBR are not isomorphic to a decrease in second-period income.[16] Rather we need to take into account this differential effect on the marginal benefits of the two instruments considered as a part of the comparative efficiency considerations. This gives rise to the following conditions:

$$MC_s^s \mathrm{Cov}\left(\widetilde{\varepsilon}, \widetilde{MB}_s^e - c' \widetilde{MB}_s^s\right) > \mathrm{Cov}\left(\widetilde{MB}_s^e, \widetilde{\varepsilon}\widetilde{MB}_s^s\right) - \mathrm{Cov}\left(\widetilde{MB}_s^s, \widetilde{\varepsilon}\widetilde{MB}_s^e\right) \tag{9}$$

$$\frac{1}{\mathrm{Cov}\left(\widetilde{\varepsilon}, \widetilde{MB}_s^s\right)}\left(\mathbb{E}\widetilde{MB}_e^s - MC_e^s\right) > \frac{1}{\mathrm{Cov}\left(\widetilde{\varepsilon}, \widetilde{MB}_s^e\right)}\left(\mathbb{E}\widetilde{MB}_e^e - MC_e^e\right). \tag{10}$$

As before the tilde indicates the presence of the background risk. The left-hand side of condition (9) is related to condition (5) for the comparative efficiency when second-period income changes.

[15] Alternatively, it can be interpreted as the risk premium in the sense of Pratt (1964) for the zero-mean risk and the utility function given by the negative of marginal utility.

[16] An exception is CARA utility for which the risk premium and the precautionary premium coincide when the risk is identical and are independent of income.

However, the right-hand side picks up the effect that background risk does not affect utility and marginal utility in the same way and measures the differential impact. (10) is a modification of (7) to account for background risk. This gives us the following proposition.

Proposition 4.b. *When the DM is prudent in the second period ($V''' > 0$), increasing the size of a SPBR*

- *leads to more risk management if and only if (9) is satisfied,*
- *leads to more savings if and only if (10) is satisfied.*

In essence, we also observe for the case of SPBR that comparative efficiency of the various instruments at hand determines when the direct or the indirect effect prevails. How to measure comparative efficiency depends on the decision context at hand and cannot be answered generically. We skip the cases of LSBR and NSBR which can be handled analogously.

4 Generalizations

The central finding of this paper is that risk management and saving are substitutes in a very general sense. As shown in the previous two sections, this substitution has important implications for trade-offs around income effects and background risks. Still, our model uses some simplifying assumptions that we address in this section. We show that the main finding can still be considerably generalized such that simplifications were mainly due to ease the exposition.

4.1 Multiple States of the World

It is common in the prevention literature to simplify the risk of loss by assuming two states of the world, a loss state and a no-loss state. Lee (1998) provides an extension of what he calls self-insurance-cum-protection decisions to multiple states of the world. We modify his approach and introduce it to our setup. Let losses be given by random variable $\widetilde{l} = \widetilde{l}(e)$ with cumulative distribution function $F(e)$. Risk management is assumed to be beneficial as it reduces the size of a loss conditional an a given state of nature; this is $l_e(e, x) \leq 0$ where x represents the state of the world. Furthermore, we include that $F_e \geq 0$ which means that an increase in risk management implies a first-order stochastic deterioration in the distribution of losses. Roughly speaking, probability mass is shifted to the left such that the probability that the realized loss be less than a threshold becomes larger whatever the threshold. As losses are subtracted from future income, this implies that a first-order stochastic improvement of future income overall.

With these specifications, the DM's objective function becomes

$$\max_{e,s} EU = U(w_1 - c(e) - s) + \beta \int V(w_2 - l(e,x) + (1+r)s)dF(e,x).$$

The relationship between the intensity of risk management and saving is governed by the cross derivative of expected utility. In the case considered here, it is given by

$$EU_{es} = c'U'' + \beta(1+r)\int V'dF_e(e,x) - \beta(1+r)\int V''l_e dF(e,x). \tag{11}$$

The first summand is negative because an increase in saving increases the marginal cost of risk management. Also the second is negative because e orders the distribution of losses according to first-order stochastic dominance. Due to risk aversion, marginal utility is diminishing so that first-order stochastic improvements are disliked. Intuitively, with higher income in the second period it pays less to reduce the likelihood of losses. Finally, also the last term is negative due to risk aversion. Consequently, the logic that risk management and saving are substitute carries over to the case of multiple state of the world. The next step is to investigate the higher order case.

4.2 Nth Order Stochastic Dominance

Rather than restricting our attention to first-order stochastic improvements in the distribution of losses, we can also investigate the general case of Nth order stochastic dominance (Ekern, 1980).[17] To this end we assume that the cumulative distribution functions are ordered by the intensity of risk management variable e,

$$\forall e_1 > e_2 : F(e_1) \succ_N F(e_2),$$

with $\succ_N$ denoting Nth order stochastic dominance. According to Ekern (1980) we know that every DM with $(-1)^k V^{(k)}(w) < 0$ for $k = 1, \ldots, N$ will unanimously prefer $F(e_1)$ to $F(e_2)$. Under the assumption that $(-1)^k V^{(k)}(w) < 0$ for $k = 1, \ldots, N+1$ we can recoup the finding that risk management and saving are substitutes. Inspecting (11) reveals that the first and the

[17] Jindapon and Neilson (2007) propose a model where the effects of risk management are not restricted to the first order.

third term are unaffected by moving to the higher-order case. For the middle term we can exploit the fact that the subsequent derivatives of function V' change sign up to the Nth derivative but start with a minus so that Nth order stochastic improvements are disliked. This indicates that the benefit from Nth oder stochastic improvements is smaller for higher income levels, i.e., when savings are larger.

4.3 Non-separable Utility

Thus far we have assumed that utility is intertemporally separable, i.e., changes in consumption at one point in time do not affect marginal utility of consumption at other points in time. To avoid confusion, we denote by lower-case $u = u(c_1, c_2)$ utility of an intertemporal consumption bundle (c_1, c_2) when this assumption is relaxed. Then, the DM's objective is given by

$$\max_{e,s} EU = p(e)u(w_1 - c(e) - s, w_2 - l(e) + (1+r)s) + (1-p(e))u(w_1 - c(e) - s, w_2 + (1+r)s),$$

and we obtain the relevant cross-derivative of expected utility with respect to risk management and saving as follows:

$$\begin{aligned} EU_{es} &= c'\left(pu_{11}(L) + (1-p)u_{11}(N)\right) - c'(1+r)\left(pu_{12}(L) + (1-p)u_{12}(N)\right) \\ &+ p'\left[u_1(N) - u_1(L)\right] + p'(1+r)\left[u_2(L) - u_2(N)\right] + pl'u_{12}(L) - pl'(1+r)u_{22}(L). \end{aligned}$$

Subscripts denote partial derivatives with respect to consumption, "L" denotes the consumption bundle associated with the second-period loss state and "N" denotes the consumption bundle associated with the second-period no-loss state. The first term is negative due first-period risk aversion, the fourth one is negative due to second-period risk aversion and the sixth one is also negative due to second-period risk aversion. The signs of the second, third and fifth depend on the cross-derivative of utility with respect to consumption today and tomorrow. They are negative if the DM is correlation-loving ($u_{12} > 0$) or correlation-neutral ($u_{12} = 0$), which corresponds to the intertemporally separable case (Eeckhoudt et al., 2007). Note that also the assumption of correlation aversion ($u_{12} < 0$) is not incompatible with the behavioral notion of substitution between risk management and saving as long as the individual is "not too" correlation-averse.[18]

[18] Technically speaking, correlation-loving and correlation-neutral preferences are sufficient for substitution between risk management and saving, whereas correlation aversion is necessary when there is complementarity between the two instruments.

5 Conclusion and Discussion

This paper studies the interplay of intertemporal risk management decisions and saving. Inspired by Lee (1998), we interpret risk management broadly. In our model, it may take the form of reducing the probability of loss, the severity of loss or both simultaneously. Although theoretical research has often analyzed self-protection and self-insurance separately – due to fundamental differences between these two instruments as related to market insurance, comparative risk aversion and income effects – real-life examples suggest to at least permit both mechanisms to be at work at the same time. Moreover, we explicitly model the intertemporal cost-benefit structure inherent in many risk management decisions. To this end, we assume expenditures to precede the benefit in time. As soon as this biperiodic perspective is taken into account, saving appears as a natural "rival" to risk management. The reason is the apparent similarity in its cost-benefit structure, which is to trade-off current consumption for changes in expected future consumption.

This intuition carries over to the formal analysis. In the benchmark model with separable utility and two states of the world we find a substitution effect between risk management and saving which extends prior literature (Dionne and Eeckhoudt, 1984; Menegatti and Rebessi, 2011). We show that this finding appears as a universal property of joint risk management and saving decisions. It carries over to cases where multiple states of the world are considered, where the effects of risk management are not restricted to 1st-order stochastic changes in the distribution of final wealth and – under suitable technical assumptions – where utility is no longer intertemporally separable. In this sense, we can establish that the substitution effect between risk management and saving is robust to a huge variety of potential extensions of the model. We study its consequences in the context of changes in income and background risk. The intuition is that, besides direct effects, substitution between the two instruments introduces an indirect effect that is typically of opposite sign as the direct one. Consequently, exogenous changes will typically have ambiguous marginal effects on the instruments at hand and conditions to resolve this ambiguity depend much on the structure of the decision problem at hand.

Although this sounds like a technical subtlety in itself, there is huge explanatory power in this simple observation. Public policy often targets individual saving and risk management behavior. This is the case for tax subsidized savings programs and investments in safety. As mentioned in the introduction, the effects on individual behavior are not always as conjectured. One potential explanation, which needs to be taken into account theoretically and empirically, is inherent in our model. Agents can be heterogeneous as related to their risk management technology and investments in risk management might (partially) be unobservable and thus hard to identify. Furthermore, the (perceived) risk exposure becomes endogenous and finally individual's (perceived) rate of return on saving is not necessarily consistent with market interest rates. Taken together, this implies that a) the direct effects of exogenous changes are hard to measure and necessarily individual-specific, and that b) the same applies to the substitution effect between the various instruments at hand to the agent. As a consequence, for the vast majority of DMs changes in behavior will be *a priori* ambiguous and – as illustrated in our paper – equivalent conditions will be required to resolve the ambiguity. As these in turn depend on preference parameters again, it is hard to judge the informativeness of empirical evaluations of public policy that use aggregate data and abstract from preference heterogeneity.

This paper is devoted to two fundamental instruments individuals use to cope with future consumption uncertainty, risk management and saving. Substitution between the two is a fundamental property with far-reaching consequences for all kinds of comparative statics analysis. We hope that our paper provides guidance to implement these cross-effects in future theoretical and empirical work to achieve a more complete picture of individual reactions to exogenous events.

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Appendix

Proof of Proposition 1: By Footnote 6, the second-order condition is globally satisfied, i.e., $EU_{ee} < 0$. Therefore, by application of the Implicit Function Theorem to (2), the direction of a marginal effect regarding an exogenous parameter is given by the respective cross-derivative. Now

$$\begin{aligned} EU_{ew_1} &= -c'U_1'' > 0, \\ EU_{ew_2} &= \beta p'(V_{2L}' - V_{2N}') - \beta p l' V_{2L}'' < 0, \\ EU_{e\beta} &= p'(V_{2L} - V_{2N}) - p l' V_{2L}' > 0, \end{aligned}$$

which completes the proof. □

Proof of Remark 1: Differentiating the first-order condition (3) with respect to saving yields

$$\frac{de}{ds} = -\frac{EU_{es}}{EU_{ee}} = -\frac{c'U_1'' + \beta p'(1+r)\left[V_{2L}' - V_{2N}'\right] - pl'V_{2L}''(1+r)}{EU_{ee}}.$$

Is is easy to see that EU_{es} is negative and so is de/ds. □

Proof of Proposition 2: Totally differentiating the system of the first-order conditions (3) and (4) with respect to w_1 yields

$$\begin{aligned} EU_{ee}de + EU_{es}ds + EU_{ew_1}dw_1 &= 0, \\ EU_{ss}ds + EU_{es}de + EU_{sw_1}dw_1 &= 0. \end{aligned}$$

From this we can solve for the marginal effects on risk management and saving,

$$\begin{aligned} \frac{de}{dw_1} &= -\frac{1}{D}\left(EU_{ss}EU_{ew_1} - EU_{es}EU_{sw_1}\right), \\ \frac{ds}{dw_1} &= -\frac{1}{D}\left(EU_{ee}EU_{sw_1} - EU_{es}EU_{ew_1}\right), \end{aligned}$$

where D is the determinant of the Hessian, $D = EU_{ee}EU_{ss} - EU_{es}^2$. It is assumed to be positive for maximality. We can now rewrite the effects in terms of marginal benefits and costs of the different activities. Note that $EU_{ew_1} = -MC_{w_1}^e = -c'U_1'' = MC_s^e$ and that $EU_{sw_1} = -MC_{w_1}^s = -U_1'' = MC_s^s$. Furthermore, as both risk management and saving imply costs only in the first period, it holds that $MC^e = c'U_1' = c'MC^s$. With these relations we can simplify

$$\begin{aligned} -EU_{ss}EU_{ew_1} + EU_{es}EU_{sw_1} &= -MC_s^e\left(-MC_s^s + MB_s^s\right) + MC_s^s\left(-MC_s^e + MB_s^e\right) \\ &= MC_s^s\left(MB_s^e - c'MB_s^s\right), \end{aligned}$$

from which we obtain condition (5), and

$$\begin{aligned} -EU_{ee}EU_{sw_1} + EU_{es}EU_{ew_1} &= -MC_s^s\left(-MC_e^e + MB_e^e\right) + MC_s^e\left(-MC_s^e + MB_s^e\right) \\ &= MC_s^s\left(c'\left(MB_s^e - MC_s^e\right) - \left(MB_e^e - MC_e^e\right)\right), \end{aligned}$$

from which we obtain condition (6). Consequently, we can conclude that de/dw_1 is positive if and only if (5) holds and that ds/dw_1 is positive if and only if (6) holds. The procedure for the other exogenous variables w_2 and β is analogous. □

Proof of Proposition 3: The idea of the proof is to evaluate the first-order expression in the presence of background risk at the optimal intensity of risk management when background risk is absent. Due to concavity of the objective function, a positive (negative) sign indicates that the introduction of background risk implies an increase (decrease) of optimal risk management. For FPBR we obtain

$$-c'\mathbb{E}U_{1\tilde{\varepsilon}}' + \beta p'[V_{2L} - V_{2N}] - \beta p l' V_{2L}' = -c'\left(\mathbb{E}U_{1\tilde{\varepsilon}}' - U_1'\right),$$

due to the first-order condition (2) without background risk. For a prudent DM marginal utility is convex and consequently $\mathbb{E}U_{1\tilde{\varepsilon}}' > U_1'$ by Jensen's inequality. Overall the sign is negative indicating a decrease in risk management.

For SPBR we obtain

$$\begin{aligned} & -c'\mathbb{E}U_1' + \beta p'[\mathbb{E}V_{2L\tilde{\varepsilon}} - \mathbb{E}V_{2N\tilde{\varepsilon}}] - \beta p l' \mathbb{E}V_{2L\tilde{\varepsilon}}' \\ = \; & -\beta p'\left[\mathbb{E}\left(V_{2N\tilde{\varepsilon}} - V_{2L\tilde{\varepsilon}}\right) - \left(V_{2N} - V_{2L}\right)\right] - \beta p l'\left(\mathbb{E}V_{2L\tilde{\varepsilon}}' - V_{2L}'\right). \end{aligned}$$

For a prudent DM, the second summand is positive according to Jensen's inequality. For the first summand observe that $V_{2N} - V_{2L}$ is a convex function of wealth when preferences exhibit prudence. Consequently, another application of Jensen's inequality demonstrates that also the first summand is positive. The net effect is an increase in the risk management activity.

Similarly, for LSBR we get

$$-c'U_1' + \beta p'[\mathbb{E}V_{2L\widetilde{\varepsilon}} - V_{2N}] - \beta p l' \mathbb{E}V'_{2L\widetilde{\varepsilon}} = \beta p' \left(\mathbb{E}V_{2L\widetilde{\varepsilon}} - V_{2L}\right) - \beta p l' \left(\mathbb{E}V'_{2L\widetilde{\varepsilon}} - V'_{2L}\right),$$

where the first bracket is negative due to risk aversion and the second is positive under prudence. Consequently, the overall sign is positive indicating an increase in the intensity of the risk management activity.

Finally, for NSBR we obtain

$$-c'U_1' + \beta p'[V_{2L} - \mathbb{E}V_{2N\widetilde{\varepsilon}}] - \beta p l' V'_{2L} = -\beta p' \left(\mathbb{E}V_{2N\widetilde{\varepsilon}} - V_{2N}\right),$$

where the bracket is negative due to risk aversion. Therefore, the sign is negative so that a reduction of risk management is optimal. □

Proof of Proposition 4.a: As in the proof of Proposition 2, we utilize the Implicit Function Theorem to obtain

$$\frac{de}{dk} = -\frac{1}{D}\left(EU_{ss}EU_{ek} - EU_{es}EU_{sk}\right),$$
$$\frac{ds}{dk} = -\frac{1}{D}\left(EU_{ee}EU_{sk} - EU_{es}EU_{ek}\right).$$

D is the determinant of the Hessian matrix which is assumed to be positive for maximality. Observe that $EU_{ek} = -\mathbb{E}\widetilde{\varepsilon}\widetilde{MC}^e_{w_1} = \mathbb{E}\widetilde{\varepsilon}\widetilde{MC}^e_s$ and that $EU_{sk} = -\mathbb{E}\widetilde{\varepsilon}\widetilde{MC}^s_{w_1} = \mathbb{E}\widetilde{\varepsilon}\widetilde{MC}^s_s$. From this it follows that the effect of background risk on the margin is nil because when $k = 0$ both $\widetilde{MC}^e_s$ and $\widetilde{MC}^s_s$ are deterministic such that $EU_{ek} = EU_{sk} = 0$. After some algebraic rearrangements, we obtain

$$\frac{de}{dk} = \frac{1}{D}\mathbb{E}\widetilde{\varepsilon}\widetilde{MC}^s_s\left(MB^e_s - c'MB^s_s\right),$$
$$\frac{ds}{dk} = \frac{1}{D}\mathbb{E}\widetilde{\varepsilon}\widetilde{MC}^s_s\left(c'(MB^s_e - \mathbb{E}\widetilde{MC}^e_s) - (MB^e_e - \mathbb{E}\widetilde{MC}^e_e)\right).$$

If the DM is prudent in the first period, then $-U''$ is a decreasing function. Consequently, for higher realizations of $\widetilde{\varepsilon}$ the corresponding realization of $\widetilde{MC}^s_s$ is lower. As a consequence the two random variables are covary negatively so that $\mathbb{E}\widetilde{\varepsilon}\widetilde{MC}^s_s = \mathrm{Cov}\left(\widetilde{\varepsilon}, \widetilde{MC}^s_s\right) < 0$. Hence, the bracketed expressions determine the overall sign of the marginal effects. □

Proof of Proposition 4.b: This is very similar to the proof of Proposition 4.a and will be omitted. □