Credit Risk and Dynamic Capital Structure Choice

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Motivation

• Increasing importance of measuring and managing credit risk
  – Basel II: capital standards based on internal models
  – Important input for risk adjusted capital allocation and RAROC calculations
• Shortcomings of existing credit risk models: borrowers’ debt levels are assumed to be constant or to change non-stochastically.
• But: borrowers’ capital structure choices are dynamic. Firms adjust leverage over time.
• This may have significant influence on credit risk.
Questions addressed:

• How can firms’ dynamic capital structure choices be integrated in a credit risk model?
• What is the effect of intertemporal capital structure choices on
  – Credit spreads of corporate debt
  – Estimated distances to default
  – Expected default frequencies
  – Credit Value-at-Risk
Major findings:

• Firms’ dynamic capital structure adjustments have significant effects on credit risk

• The dynamics of capital structure adjustments
  – generally increase fair credit spreads and the expected default frequencies
  – Imply a non-monotonic relationship between distance to default and expected default frequencies.
  – When using historic data to map distance to default estimates into expected default frequencies one should separate the sample according to
    • Volatilities
    • Effective corporate tax rates
    • Estimated bankruptcy costs.
    • Expected firm growth
    • Existing bond indentures
Relevant literature

- Literature on pricing of risky corporate bonds:
  
  Without recapitalization:
  
  - Merton (JF 74)
  - Leland (JF 94, JF 96)
  - Longstaff, Schwartz (JF 95)
  - Duffie, Lando (Econ. forthc.)
  - Jarrow, Turnbull (JF 95)

  With recapitalization:
  
  - Fischer, Heinkel, Zechner (JF 89)
  - Anderson, Sundaresan (RFS 96)
  - Leland (JF 98)
  - Christensen et al (working paper 00)
Option-based credit risk models (1)

• Weakness of CreditMetrics/CreditVaR I: Transition probabilities are based on historical default frequencies and rating migration.
• Assumes that all firms in a rating class have the same default probability.
• Assumes basically that future default rates equal historical averages.
• → Backward looking.
Option-based credit risk models (2)

- KMV approach: Is firm-specific.
- Recognizes that credit risk is due to stochastic changes in the asset value of the debtor.

\[ V_t = V_0 \exp \left( \left( \mu - \frac{\sigma^2}{2} \right) T + \sigma \sqrt{T} \, Z_T \right) \]

\[ E(V_T) = V_0 e^{\mu T} \]

Source: Crouhy et al., JBF 2000
Option-based credit risk models (3)

- How to estimate firm asset value, $V_A$, and firm volatility, $\sigma_A$?
- For traded firms we can observe the value of equity and its standard deviation, $V_E$, and $\sigma_E$.
- But equity can be seen as a contingent claim on the value of the firm’s assets:
  
  $$V_E = f(V_A, \sigma_A, B, i, r)$$
  $$\sigma_E = g(V_A, \sigma_A, B, i, r)$$

- Where $B$, $i$, and $r$ denote the face value of debt, the coupon rate and the riskless rate of interest, respectively.
- Since $B$, $i$, $r$, $V_E$ and $\sigma_E$ are observable, one can back out $V_A$ and $\sigma_A$. 
Option-based credit risk models (4)

- Calculation of the distance to default (DD):
- $B_s =$ face value of short-term debt
- $B_l =$ face value of long-term debt
- Default point $= B_s + B_l / 2$

$$DD = \frac{E[V_T] - (B_s + 0.5B_l)}{\sigma_A}$$
Option-based credit risk models (5)

- From the DD one can calculate theoretical expected default frequencies: E.g. DD=2.33 → theoretical default frequency = 1%.
- KMV: maps historical DDs to actual defaults for a given risk horizon:

Source: Crouhy et al., JBF 2000
Option-based credit risk models (6)

- Main observations:
  - EDF do not converge to zero as implied by DD
  - KMV approach: does not take dynamics of borrowers’ financial decisions into account
  - In reality firms may issue additional debt or reduce debt before the risk horizon
  - Firms financing decisions will depend on the development of $V_A$. 
Option-based credit risk models (7)

What are the implications of these dynamics on credit risk?
The model (1)

Notation

\( c_t \) ........ a firm’s instantaneous free cash flow after corporate tax

\( c_t \) follows the process:

\[
\frac{dc_t}{c_t} = \mu \, dt + \sigma \, dW
\]

\( \mu \) ........ expected rate of change of \( c_t \)

\( \sigma \) ........ risk (standard deviation) of changes in \( c_t \)

\( dW \) ...... increment to a standard Wiener process
The model (2)

- Define the firm’s inverse leverage, \( y_t \), as

\[
    y_t = \frac{c_t}{r(1-\tau_p) - \hat{\mu}} - \frac{r(1-\tau_p) - \hat{\mu}}{B} \Rightarrow \frac{dy_t}{y_t} = \mu dt + \sigma dW
\]

- \( \hat{\mu} \) = risk adjusted drift of the cash flow process
- B = face value of debt
- \( r(1-\tau_p) \)......interest rate of a riskfree asset, after personal tax
The model (3)

- The value of a firm’s debt and equity are contingent claims on the inverse leverage, \(y\) and the face value of debt, \(B\):
\[
E = E(y, B), \quad D = D(y, B)
\]

- These claims must follow the differential equations:
\[
\frac{1}{2} \sigma^2 y^2 D_{yy} + \hat{\mu} y D_y - r(1 - \tau_p)D + (1 - \tau_p)iB = 0
\]
\[
\frac{1}{2} \sigma^2 y^2 E_{yy} + \hat{\mu} y E_y - r(1 - \tau_p)E - (1 - \tau_c)iB + c = 0
\]

- where \(i\) denotes the coupon rate and \(\tau_c\) is the corporate tax rate.
The model (4)

These differential equations have the following solutions:

\[ E(y, B) = B E_1 y^{m_1} + B E_2 y^{m_2} - \frac{(1 - \tau_c)i}{(1 - \tau_p)r} B + yB \]

\[ D(y, B) = B D_1 y^{m_1} + B D_2 y^{m_2} + \frac{i}{r} B \]

\[ m_{1/2} = \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \pm \sqrt{\left( \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \right)^2 + \frac{2r(1 - \tau_p)}{\sigma^2}} \]
Dynamic leverage adjustments

• If the leverage ratio $1/y$ reaches a lower critical value, $1/\bar{y}$, then the firm repurchases existing debt at $B(1+\lambda)$ and issues additional debt.

• If the leverage ratio $1/y$ reaches an upper critical value, $1/y$ then default occurs and the debtholders receive the firm’s assets after paying proportional bankruptcy costs $g$.

• When a firm issues debt, then it must pay transactions costs $k$, proportional to the new face value of debt.
Typical path for \( y \) with recapitalization
Boundary conditions

\[ E(y, B) = 0 \]
\[ E(y, B) = [E(y_0^*, B \frac{y}{y_0}) + B \frac{y}{y_0} (1-k)] - (1 + \lambda)B \]
\[ D(y, B) = [E(y_0^*, B \frac{y}{y_0}) + B \frac{y}{y_0} (1-k)](1 - g) \]
\[ D(y, B) = B(1 + \lambda) \]

Issue at par condition:
• Choose \( i \) such that:
\[ D(y_0^*, B) = B \]
Boundary conditions no recap.

• The model can also be solved for the case where recapitalizations are not allowed.

• The only differences are the boundary conditions.

\[
\lim_{y \to \infty} E(y, B) = -\frac{(1-\tau_c)i}{(1-\tau_p)r} B + yB
\]

\[
\lim_{y \to \infty} D(y, B) = \frac{i}{r} B
\]
Endogenous bankruptcy

When $y$ can be chosen by the equityholders, the following "low contact" or “smooth pasting” condition must hold (Merton 73): $E_y(y) = 0$
Debt value as a function of leverage

\[ D(1/y) \]
Equity value as a function of leverage
Numerical results (1)

base case parameters

$r = 5\%$
$
\tau_p = 35\%$
$
\tau_c = 50\%$
$
\sigma^2_y = 5\%$
$k = 1\%$
$g = 25\%$

recapitalization

$1/y_o = 58\%$
$1/y = 208\%$
$1/\bar{y} = 39\%$
i($y_o$) = 7.7\%

no recapitalization

$1/y_o = 70\%$
$1/y = 205\%$
$1/\bar{y} = 0\%$
i($y_o$) = 7.4\%
Numerical results (2)

<table>
<thead>
<tr>
<th>$\sigma^2_y$</th>
<th>1/y₀(R)</th>
<th>i*(R)</th>
<th>1/y₀(NR)</th>
<th>i*(NR)</th>
<th>Δ(1/y₀)</th>
<th>Δi*</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>58%</td>
<td>7.7%</td>
<td>70%</td>
<td>7.4%</td>
<td>-12%</td>
<td>30bp</td>
</tr>
<tr>
<td>0.04</td>
<td>60.6%</td>
<td>7.3%</td>
<td>71.8%</td>
<td>7.06%</td>
<td>-11.2%</td>
<td>24bp</td>
</tr>
<tr>
<td>0.02</td>
<td>67.9%</td>
<td>6.35%</td>
<td>77.9%</td>
<td>6.23%</td>
<td>-10%</td>
<td>12bp</td>
</tr>
<tr>
<td>$\tau_c\cdot\tau_p$</td>
<td>1/y₀(R)</td>
<td>i*(R)</td>
<td>1/y₀(NR)</td>
<td>i*(NR)</td>
<td>Δ(1/y₀)</td>
<td>Δi*</td>
</tr>
<tr>
<td>0.15</td>
<td>58%</td>
<td>7.7%</td>
<td>70%</td>
<td>7.4%</td>
<td>-12%</td>
<td>30bp</td>
</tr>
<tr>
<td>0.11</td>
<td>45%</td>
<td>7.75%</td>
<td>56%</td>
<td>7.44%</td>
<td>-11%</td>
<td>31bp</td>
</tr>
<tr>
<td>0.05</td>
<td>22%</td>
<td>5.9%</td>
<td>30%</td>
<td>5.9%</td>
<td>-8%</td>
<td>~0bp</td>
</tr>
</tbody>
</table>
Numerical results (3)

<table>
<thead>
<tr>
<th>k</th>
<th>1/y₀(R)</th>
<th>i*(R)</th>
<th>1/y₀(NR)</th>
<th>i*(NR)</th>
<th>Δ(1/y₀)</th>
<th>Δi*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>58.6%</td>
<td>7.7%</td>
<td>70%</td>
<td>7.4%</td>
<td>-11.4%</td>
<td>30bp</td>
</tr>
<tr>
<td>2%</td>
<td>58%</td>
<td>7.57%</td>
<td>68%</td>
<td>7.35%</td>
<td>-10%</td>
<td>22bp</td>
</tr>
<tr>
<td>4%</td>
<td>55.5%</td>
<td>7.26%</td>
<td>65%</td>
<td>7.18%</td>
<td>-9.5%</td>
<td>8bp</td>
</tr>
<tr>
<td>g</td>
<td>1/y₀(R)</td>
<td>i*(R)</td>
<td>1/y₀(NR)</td>
<td>i*(NR)</td>
<td>Δ(1/y₀)</td>
<td>Δi*</td>
</tr>
<tr>
<td>20%</td>
<td>65%</td>
<td>8.05%</td>
<td>75.8%</td>
<td>7.62%</td>
<td>-10.8%</td>
<td>43bp</td>
</tr>
<tr>
<td>25%</td>
<td>58.6%</td>
<td>7.7%</td>
<td>70%</td>
<td>7.4%</td>
<td>-11.4%</td>
<td>30bp</td>
</tr>
<tr>
<td>30%</td>
<td>53.6%</td>
<td>7.53%</td>
<td>65%</td>
<td>7.3%</td>
<td>-11.4%</td>
<td>23bp</td>
</tr>
</tbody>
</table>
Numerical results (4)

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$1/y_0(R)$</th>
<th>$i^*(R)$</th>
<th>$1/y_0(NR)$</th>
<th>$i^*(NR)$</th>
<th>$\Delta(1/y_0)$</th>
<th>$\Delta i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2%</td>
<td>54.7%</td>
<td>8.56%</td>
<td>66.7%</td>
<td>8.39%</td>
<td>-12%</td>
<td>17bp</td>
</tr>
<tr>
<td>0%</td>
<td>58.6%</td>
<td>7.70%</td>
<td>70%</td>
<td>7.4%</td>
<td>-11.4%</td>
<td>30bp</td>
</tr>
<tr>
<td>2%</td>
<td>74%</td>
<td>7.04%</td>
<td>74%</td>
<td>6.67%</td>
<td>~0%</td>
<td>37bp</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$1/y_0(R)$</td>
<td>$i^*(R)$</td>
<td>$1/y_0(NR)$</td>
<td>$i^*(NR)$</td>
<td>$\Delta(1/y_0)$</td>
<td>$\Delta i^*$</td>
</tr>
<tr>
<td>0%</td>
<td>58.6%</td>
<td>7.70%</td>
<td>70%</td>
<td>7.4%</td>
<td>-11.4%</td>
<td>30bp</td>
</tr>
<tr>
<td>5%</td>
<td>61.5%</td>
<td>7.40%</td>
<td>70%</td>
<td>7.4%</td>
<td>-8.5%</td>
<td>0bp</td>
</tr>
<tr>
<td>10%</td>
<td>63.8%</td>
<td>7.28%</td>
<td>70%</td>
<td>7.4%</td>
<td>-6.2%</td>
<td>-12bp</td>
</tr>
</tbody>
</table>
Summary

• Recapitalization decreases the optimal initial leverage ratio.

• Recapitalization generally increases credit spreads.

• These effects are stronger for high-risk, high corporate tax and high-growth firms and less pronounced for firms with high costs of recapitalization and high bankruptcy costs.
Model risk

• Previous results assume that $y$ and $\sigma_y$ are observable.

• In practice: $E$ and $\sigma_E$ are observable and $y$ and $\sigma_y$ must be inferred from the valuation model.

• What is the error due to using a static (=Merton type) valuation model rather than a model allowing for capital structure adjustments?
Numerical results: Model risk (1)

What is the effect of using the “wrong” Merton-type no-recap model?

Observe $E$ and $\sigma_E$; back out $y$ and $\sigma_y$; calculate the fair credit spread:

<table>
<thead>
<tr>
<th>$\sigma^2_y$</th>
<th>$i^*_{(recap)}$</th>
<th>$i^*_{(no recap)}$</th>
<th>$\Delta i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>6.35%</td>
<td>6.04%</td>
<td>31bp</td>
</tr>
<tr>
<td>0.04</td>
<td>7.30%</td>
<td>6.73%</td>
<td>43bp</td>
</tr>
<tr>
<td>0.06</td>
<td>8.14%</td>
<td>7.34%</td>
<td>60bp</td>
</tr>
<tr>
<td>0.08</td>
<td>8.85%</td>
<td>7.85%</td>
<td>100bp</td>
</tr>
</tbody>
</table>
Numerical results: Model risk (2)

What is the effect of using the “wrong” Merton-type no-recap model?

Observe $E$ and $\sigma_E$; back out $y$ and $\sigma_y$; calculate the fair credit spread:

<table>
<thead>
<tr>
<th>$\lambda$ (=call premium)</th>
<th>$i^*_{\text{(recap)}}$</th>
<th>$i^*_{\text{(no recap)}}$</th>
<th>$\Delta i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>7.75%</td>
<td>7.07%</td>
<td>71bp</td>
</tr>
<tr>
<td>5%</td>
<td>7.40%</td>
<td>7.06%</td>
<td>34bp</td>
</tr>
<tr>
<td>10%</td>
<td>7.28%</td>
<td>7.10%</td>
<td>18bp</td>
</tr>
<tr>
<td>25%</td>
<td>7.22%</td>
<td>7.20%</td>
<td>2bp</td>
</tr>
</tbody>
</table>
Numerical results: Model risk (3)

What is the effect of using the “wrong” Merton-type no-recap model?

Observe $E$ and $\sigma_E$; back out $y$ and $\sigma_y$; calculate the fair credit spread:

<table>
<thead>
<tr>
<th>$\tau_c - \tau_p$</th>
<th>$i^*$ (recap)</th>
<th>$i^*$ (no recap)</th>
<th>$\Delta i^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>7.75%</td>
<td>7.07%</td>
<td>71bp</td>
</tr>
<tr>
<td>11%</td>
<td>7.03%</td>
<td>6.92%</td>
<td>11bp</td>
</tr>
<tr>
<td>5%</td>
<td>5.93%</td>
<td>5.97%</td>
<td>-4bp</td>
</tr>
</tbody>
</table>
Summary

• Generally, using a „static“ option pricing model to infer asset risk leads to unerestimation of fair credit spreads.

• This underestimation is
  – more severe for high-risk firms and for firms with high effective corporate tax rates
  – less severe for firms with high costs of recapitalization.
Numerical results: Expected default frequencies

- Without recapitalizations: theoretical expected default probabilities (TEDF) converge to zero with DD.
- With recapitalizations, TEDF are non-monotone in DD.
Numerical results: Expected default frequencies

- Expected default frequency and frequency of a recapitalization with subsequent default:
- For large DD, the remaining default risk is only due to corporate leverage changes.
DD and Expected default frequencies

- The graph below displays the understimation of expected default probabilities for various risk levels.
DD and Expected default frequencies

- The graph below displays the understimation of expected default probabilities for various expected cash flow growth rates.
Conclusions (1)

- Capital structure dynamics have important effect on credit risk.
- Traditional DD measure is not a sufficient statistic for true credit risk.
- Mapping from DD into EDF is u-shaped.
- Capital structure dynamics may explain the slow convergence of empirical EDF’s to zero.
Conclusions (2)

• Estimating the empirical relationship between DD and expected default frequencies requires conditioning on:
  – firms’ volatilities,
  – expected growth,
  – restrictions in existing bond indentures
  – firms’ effective corporate tax rates.
Possible extensions

- Consideration of alternative bankruptcy criteria
- Empirical tests
- Allow for multiple debt issues
- Modelling other motives for optimal capital structure (agency considerations etc.)