Credit Risk and Dynamic Capital Structure Choice

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Motivation

- Increasing importance of measuring and managing credit risk
  - Basel II: capital standards based on internal models
  - Important input for risk adjusted capital allocation and RAROC calculations
- Shortcomings of existing credit risk models: borrowers' debt levels are assumed to be constant or to change non-stochastically.
- But: borrowers' capital structure choices are dynamic. Firms adjust leverage over time.
- This may have significant influence on credit risk.

Questions addressed:

- How can firms' dynamic capital structure choices be integrated in a credit risk model?
- What is the effect of intertemporal capital structure choices on
  - Credit spreads of corporate debt
  - Estimated distances to default
  - Expected default frequencies
  - Credit Value-at-Risk

Major findings:

- Firms' dynamic capital structure adjustments have significant effects on credit risk
- The dynamics of capital structure adjustments
  - generally increase fair credit spreads and the expected default frequencies
  - Imply a non-monotonic relationship between distance to default and expected default frequencies.
  - When using historic data to map distance to default estimates into expected default frequencies one should separate the sample according to
    - Volatilities
    - Effective corporate tax rates
    - Estimated bankruptcy costs.
    - Expected firm growth
    - Existing bond indentures

Relevant literature

- Literature on pricing of risky corporate bonds:

  Without recapitalization:
  - Merton (JF 74)
  - Leland (JF 94, JF 96)
  - Longstaff, Schwartz (JF 95)
  - Duffie, Lando (Econ. forthc.)
  - Jarrow, Turnbull (JF 95)

  With recapitalization:
  - Fischer, Heinkel, Zechner (JF 89)
  - Anderson, Sundaresan (RFS 96)
  - Leland (JF 98)
  - Christensen et al (working paper 00)

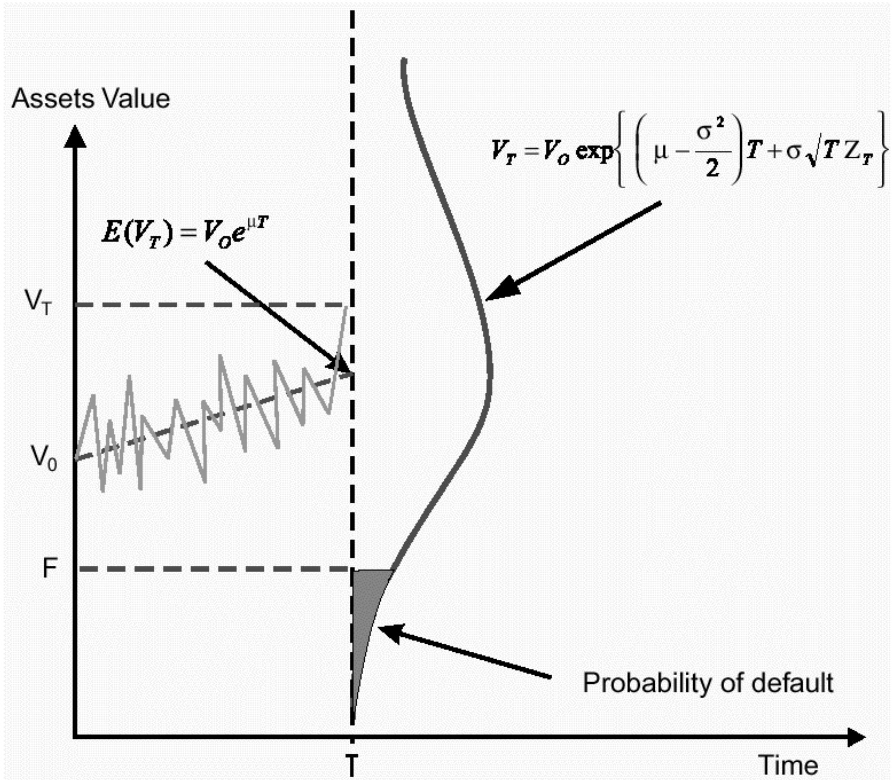
Option-based credit risk models (1)

- Weakness of CreditMetrics/CreditVaR I: Transition probabilities are based on historical default frequencies and rating migration.
- Assumes that all firms in a rating class have the same default probability.
- Assumes basically that future default rates equal historical averages.
- → Backward looking.

Option-based credit risk models (2)

- KMV approach: Is firm-specific.

- Recognizes that credit risk is due to stochastic changes in the asset value of the debtor.



Source: Crouhy et al., JBF 2000

Option-based credit risk models (3)

- How to estimate firm asset value,V_A, and firm volatility, σ_A?
- For traded firms we can observe the value of equity and its standard deviation, $V_{E,}$ and σ_E
- But equity can be seen as a contingent claim on the value of the firm's assets:

$$V_E = f(V_A, \sigma_A, B, i, r)$$
$$\sigma_E = g(V_A, \sigma_A, B, i, r)$$

- Where B, i, and r denote the face value of debt, the coupon rate and the riskless rate of interest, respectively.
- Since B, i, r, V_E and σ_E are observable, one can back out V_A and σ_A

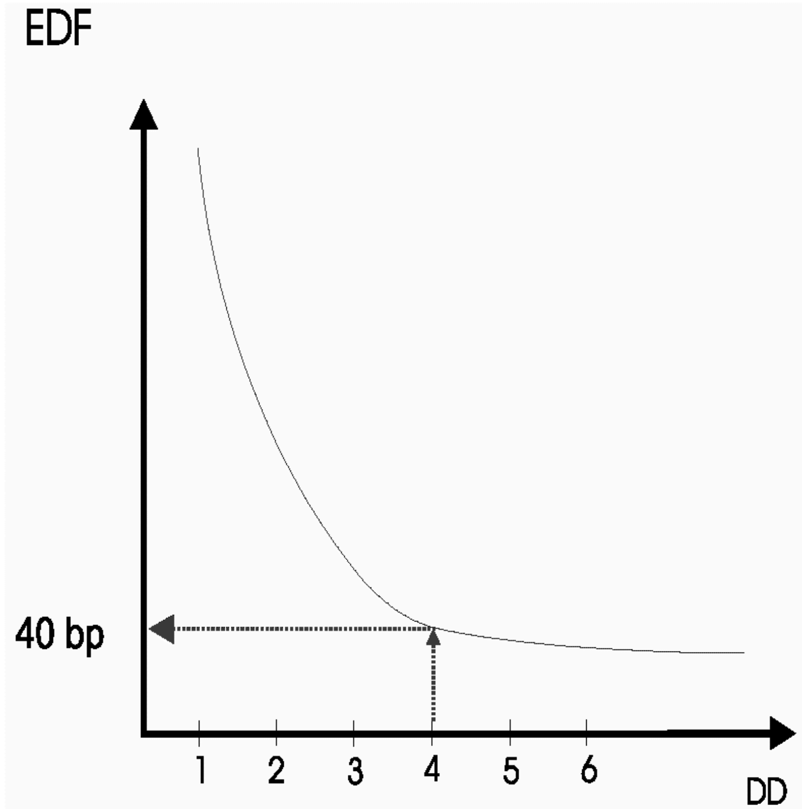
Option-based credit risk models (4)

- Calculation of the distance to default (DD):
- B_s =face value of short-term debt
- B_l =face value of long-term debt
- Default point= $B_s + B_l/2$

$$DD = \frac{E[V_T] - (B_s + 0.5B_l)}{\sigma_A}$$

Option-based credit risk models (5)

- From the DD one can calculate theoretical expected default frequencies: E.g. DD=2,33→ theoretical default frequency = 1%.
- KMV: maps historical DDs to actual defaults for a given risk horizon:

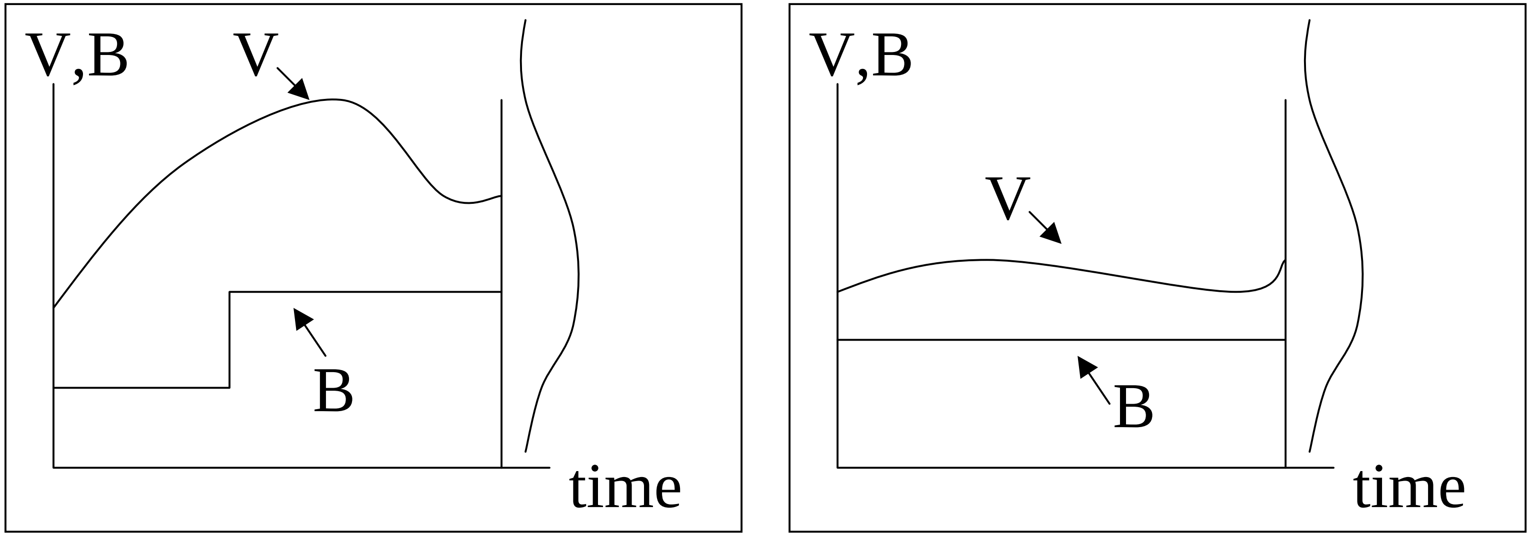


Source: Crouhy et al., JBF 2000

Option-based credit risk models (6)

- Main observations:
  - EDF do not converge to zero as implied by DD
  - KMV approach: does not take dynamics of borrowers' financial decisions into account
  - In reality firms may issue additional debt or reduce debt before the risk horizon
  - Firms financing decisions will depend on the development of V_A.

Option-based credit risk models (7)



What are the implications of these dynamics on credit risk?

The model (1)

Notation

c_t ....a firm's instantaneous free cash flow after corporate tax

c_t follows the process: $$\frac{dc_t}{c_t} = \mu\, dt + \sigma\, dW$$

μ .... expected rate of change of c_t

σ .... risk (standard deviation) of changes in c_t

dW ...... increment to a standard Wiener process

The model (2)

- Define the firm's inverse leverage, y_t, as

$$y_t = \frac{\dfrac{c_t}{r(1-\tau_p)-\hat{\mu}}}{B} \Rightarrow \qquad \frac{dy_t}{y_t} = \mu\, dt + \sigma\, dW$$

- $\hat{\mu}$ =risk adjusted drift of the cash flow process
- B=face value of debt
- r (1- τ_p)......interest rate of a riskfree asset, after personal tax

The model (3)

- The value of a firm's debt and equity are contingent claims on the inverse leverage, y and the face value of debt, B: $E = E(y, B)$, $D = D(y, B)$
- These claims must follow the differential equations:

$$\tfrac{1}{2}\sigma^2 y^2 D_{yy} + \hat{\mu} y D_y - r(1-\tau_P)D + (1-\tau_P)iB = 0$$
$$\tfrac{1}{2}\sigma^2 y^2 E_{yy} + \hat{\mu} y E_y - r(1-\tau_P)E - (1-\tau_C)iB + c = 0$$

- where i denotes the coupon rate and τ_c is the corporate tax rate.

The model (4)

- These differential equations have the following solutions:

$$E(y,B) = B\,E_1 y^{m1} + B\,E_2 y^{m2} - \frac{(1-\tau_C)i}{(1-\tau_P)r} B + yB$$

$$D(y,B) = B\,D_1 y^{m1} + B\,D_2 y^{m2} + \frac{i}{r} B$$

$$m_{1/2} = \frac{1}{2} - \frac{\hat{\mu}}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\hat{\mu}}{\sigma^2}\right)^2 + \frac{2r(1-\tau_P)}{\sigma^2}}$$

Dynamic leverage adjustments

- If the leverage ratio 1/y reaches a lower critical value, $1/\bar{y}$, then the firm repurchases existing debt at B $(1+\lambda)$ and issues additional debt.

- If the leverage ratio 1/y reaches an upper critical value, $1/\underline{y}$ then default occurs and the debtholders receive the firm‘s assets after paying proportional bankruptcy costs g.

- When a firm issues debt, then it must pay transactions costs k, proportional to the new face value of debt.

Typical path for y with recapitalization

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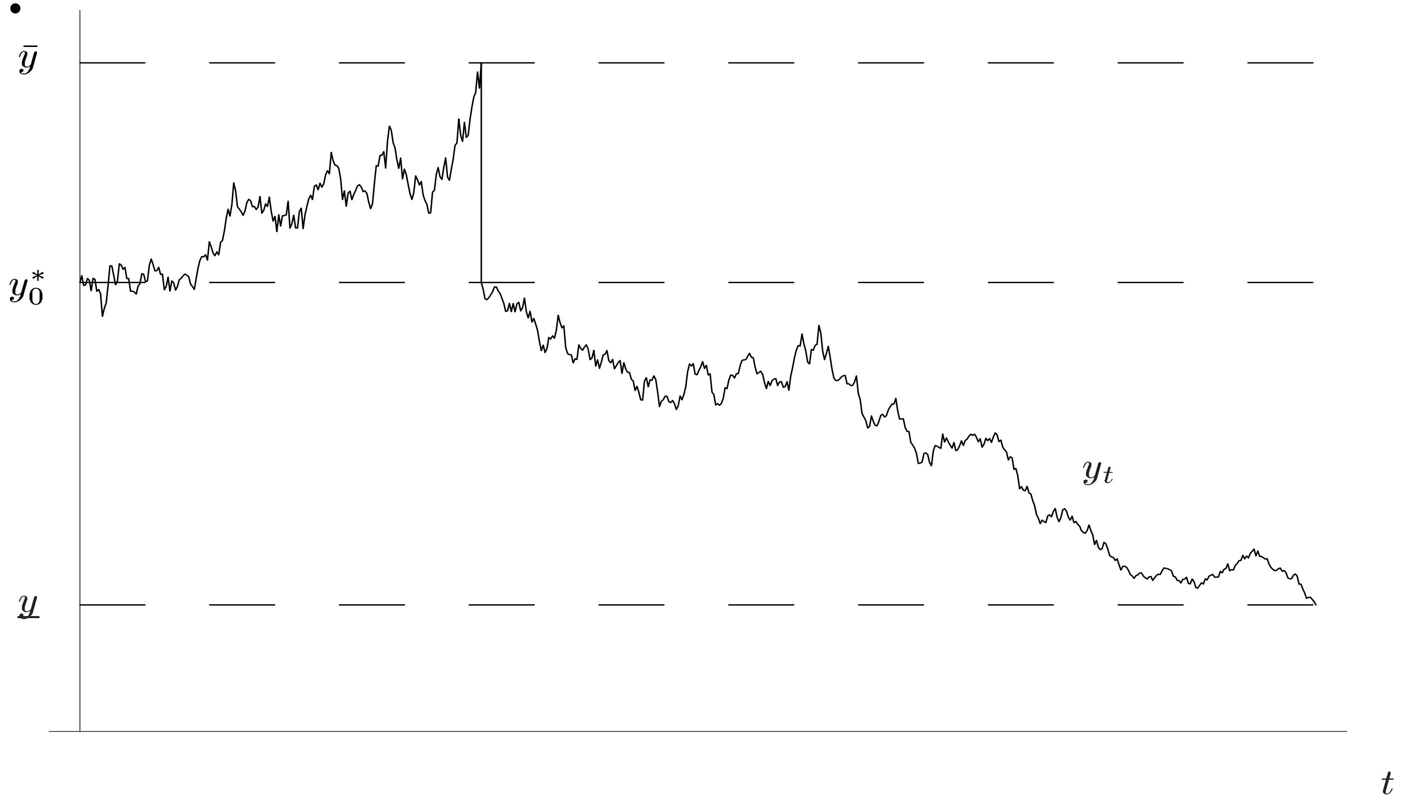


Boundary conditions

$$E(\underline{y}, B) = 0$$

$$E(\bar{y}, B) = [E(y_0^*, B\frac{\bar{y}}{y_0^*}) + B\frac{\bar{y}}{y_0^*}(1-k)] - (1+\lambda)B$$

$$D(\underline{y}, B) = [E(y_0^*, B\frac{\underline{y}}{y_0^*}) + B\frac{\underline{y}}{y_0^*}(1-k)](1-g)$$

$$D(\bar{y}, B) = B(1+\lambda)$$

Issue at par condition:

- Choose i such that:

$$D(y_0^*, B) = B$$

Boundary conditions no recap.

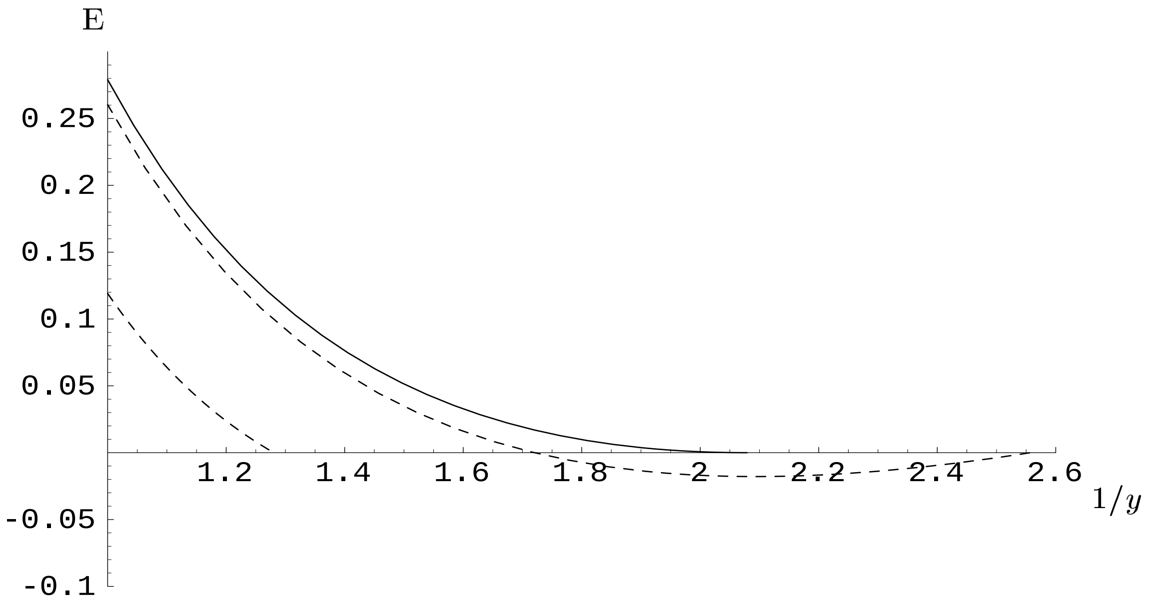
- The model can also be solved for the case where recapitalizations are not allowed.
- The only differences are the boundary conditions.

$$\lim_{y\to\infty} E(y,B) = -\frac{(1-\tau_C)i}{(1-\tau_P)r}B + yB$$

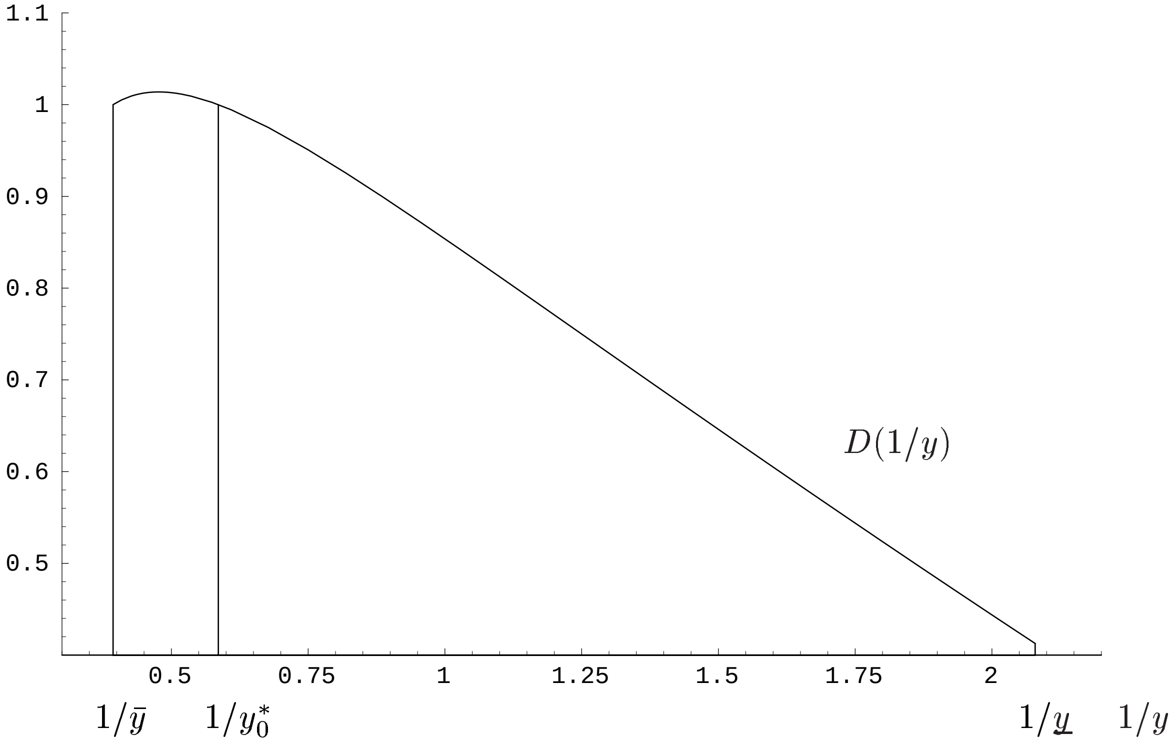
$$\lim_{y\to\infty} D(y,B) = \frac{i}{r}B$$

Endogenous bankruptcy

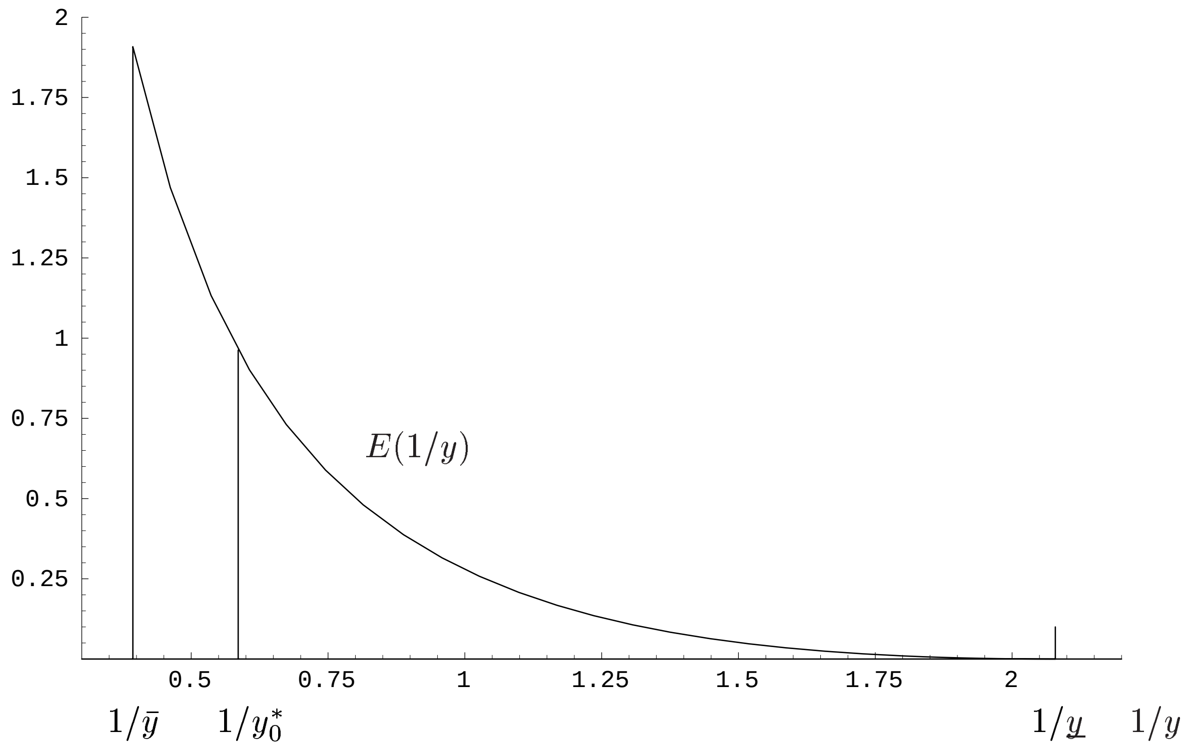
When $\underline{y}$ can be chosen by the equityholders, the following "low contact" or "smooth pasting" condition must hold (Merton 73): $E_y(\underline{y}) = 0$



Debt value as a function of leverage



Equity value as a function of leverage



Numerical results (1)

base case parameters

$r = 5\ \%$

$\tau_p = 35\%$

$\tau_c = 50\%$

$\sigma^2_y = 5\ \%$

$k = 1\%$

$g = 25\%$

recapitalization

$1/y_o = 58\%$

$1/\underline{y} = 208\%$

$1/\bar{y} = 39\%$

$i(y_o) = 7.7\%$

no recapitalization

$1/y_o = 70\%$

$1/\underline{y} = 205\%$

$1/\bar{y} = 0\%$

$i(y_o) = 7.4\%$

Numerical results (2)

σ^2_y	$1/y_0(R)$	$i^*(R)$	$1/y_0(NR)$	$i^*(NR)$	$\Delta(1/y_o)$	Δi^*
0.05	58%	7.7%	70%	7.4%	-12%	30bp
0.04	60.6%	7.3%	71.8%	7.06%	-11.2%	24bp
0.02	67.9%	6.35%	77.9%	6.23%	-10%	12bp
$\tau_{c-}\tau_p$	$1/y_0(R)$	$i^*(R)$	$1/y_0(NR)$	$i^*(NR)$	$\Delta(1/y_o)$	Δi^*
0.15	58%	7.7%	70%	7.4%	-12%	30bp
0.11	45%	7.75%	56%	7.44%	-11%	31bp
0.05	22%	5.9%	30%	5.9%	-8%	~0bp

Numerical results (3)

k	**$1/y_0$(R)**	**i^*(R)**	**$1/y_0$(NR)**	**i^*(NR)**	**$\Delta(1/y_o)$**	**Δi^***
1%	58.6%	7.7%	70%	7.4%	-11.4%	30bp
2%	58%	7.57%	68%	7.35%	-10%	22bp
4%	55.5%	7.26%	65%	7.18%	-9.5%	8bp
g	**$1/y_0$(R)**	**i^*(R)**	**$1/y_0$(NR)**	**i^*(NR)**	**$\Delta(1/y_o)$**	**Δi^***
20%	65%	8.05%	75.8%	7.62%	-10.8%	43bp
25%	58.6%	7.7%	70%	7.4%	-11.4%	30bp
30%	53.6%	7.53	65%	7.3%	-11.4%	23bp

Numerical results (4)

$\hat{\mu}$	**$1/y_0$(R)**	**i^*(R)**	**$1/y_0$(NR)**	**i^*(NR)**	**$\Delta(1/y_o)$**	**Δi^***
-2%	54.7%	8.56%	66.7%	8.39%	-12%	17bp
0%	58.6%	7.70%	70%	7.4%	-11.4%	30bp
2%	74%	7.04%	74%	6.67%	~0%	37bp
λ	**$1/y_0$(R)**	**i^*(R)**	**$1/y_0$(NR)**	**i^*(NR)**	**$\Delta(1/y_o)$**	**Δi^***
0%	58.6%	7.70%	70%	7.4%	-11.4%	30bp
5%	61.5%	7.40%	70%	7.4%	-8.5%	0bp
10%	63.8%	7.28%	70%	7.4%	-6.2%	-12bp

Summary

- Recapitalization decreases the optimal initial leverage ratio.
- Recapitalization generally increases credit spreads.
- These effects are stronger for high-risk, high corporate tax and high-growth firms and less pronounced for firms with high costs of recapitalization and high bankruptcy costs.

Model risk

- Previous results assume that y and σ_y are observable.
- In practice: E and σ_E are observable and y and σ_y must be inferred from the valuation model.
- What is the error due to using a static (=Merton type) valuation model rather than a model allowing for capital structure adjustments?

Numerical results: Model risk (1)

What is the effect of using the “wrong” Merton-type no-recap model?

Observe E and $\sigma_{E;}$ back out y and $\sigma_{y;}$ calculate the fair credit spread:

σ^2_y	$i^*_{(recap)}$	$i^*_{(no\ recap)}$	Δi^*
0.02	6.35%	6.04%	31bp
0.04	7.30%	6.73%	43bp
0.06	8.14%	7.34%	60bp
0.08	8.85%	7.85%	100bp

Numerical results: Model risk (2)

What is the effect of using the “wrong” Merton-type no-recap model?

Observe E and $\sigma_{E;}$ back out y and $\sigma_{y;}$ calculate the fair credit spread:

λ (=call premium)	**$i^*_{(recap)}$**	**$i^*_{(no\ recap)}$**	**$\Delta\, i^*$**
0%	7,75%	7,07%	71bp
5%	7,40%	7,06%	34bp
10%	7,28%	7,10%	18bp
25%	7,22%	7,20%	2bp

Numerical results: Model risk (3)

What is the effect of using the “wrong” Merton-type no-recap model?

Observe E and $\sigma_{E;}$ back out y and $\sigma_{y;}$ calculate the fair credit spread:

$\tau_c - \tau_p$	$i^*_{(recap)}$	$i^*_{(no\ recap)}$	$\Delta\, i^*$
15%	7,75%	7,07%	71bp
11%	7.03%	6.92%	11bp
5%	5.93%	5.97%	-4bp

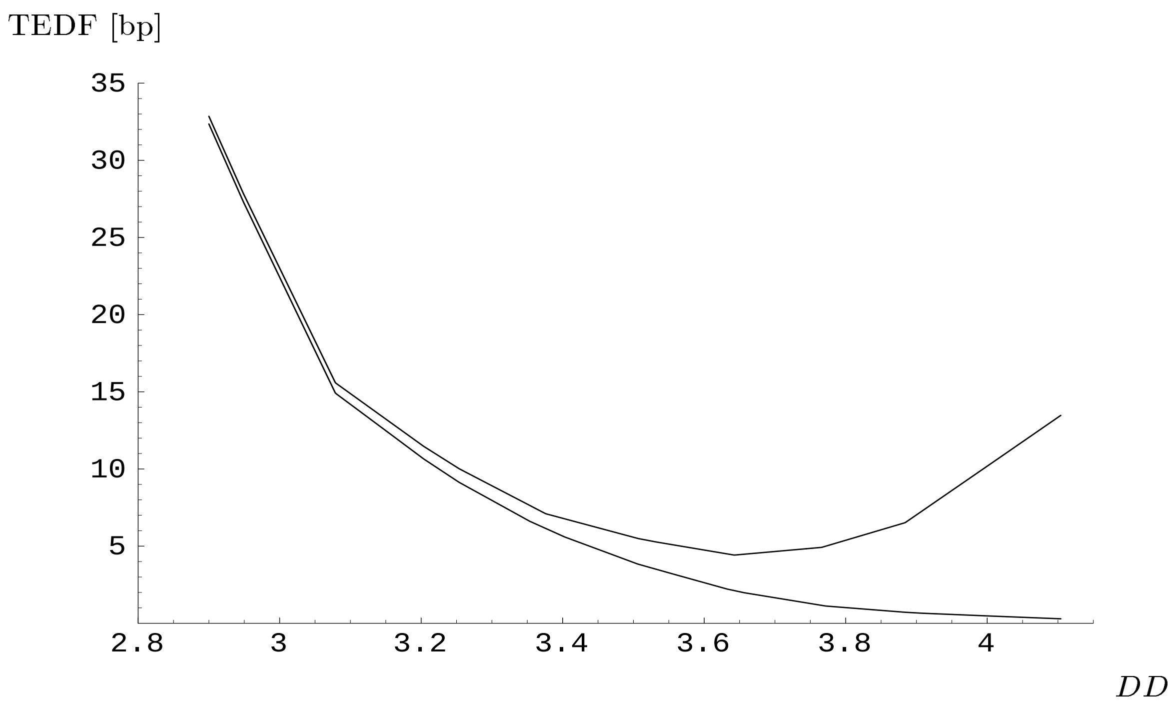
Summary

- Generally, using a „static“ option pricing model to infer asset risk leads to unerestimation of fair credit spreads.

- This underestimation is
  - more severe for high-risk firms and for firms with high effective corporate tax rates
  - less severe for firms with high costs of recapitalization.

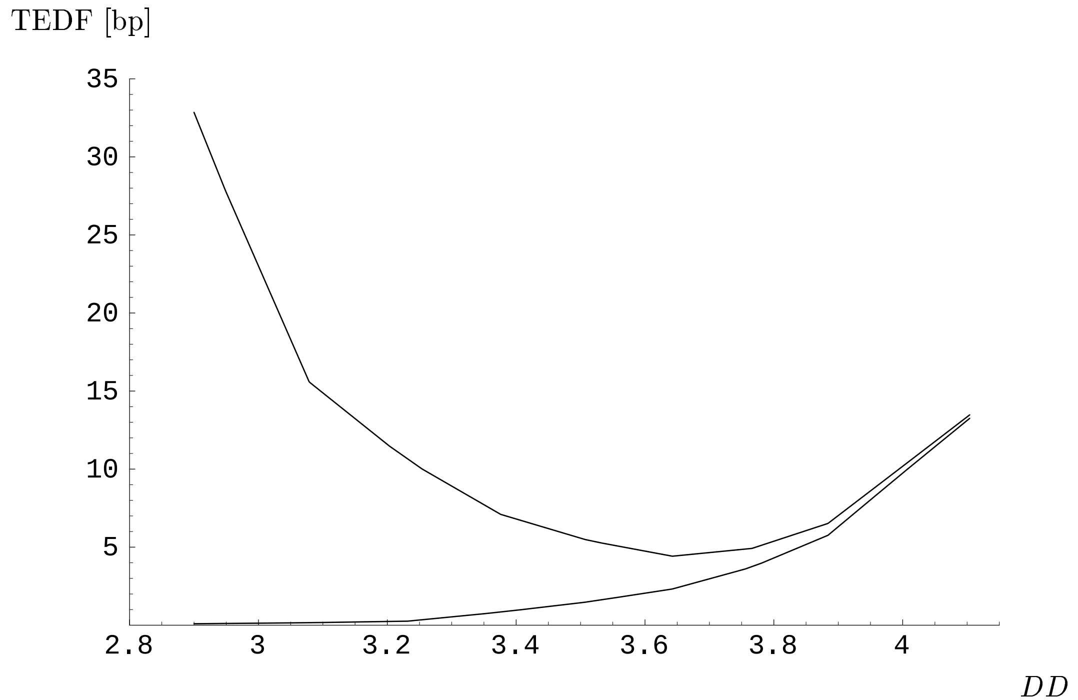
Numerical results: Expected default frequencies

- Without recapitalizations: theoretical expected default probabilities (TEDF) converge to zero with DD.
- With recapitalizations, TEDF are non-monotone in DD.



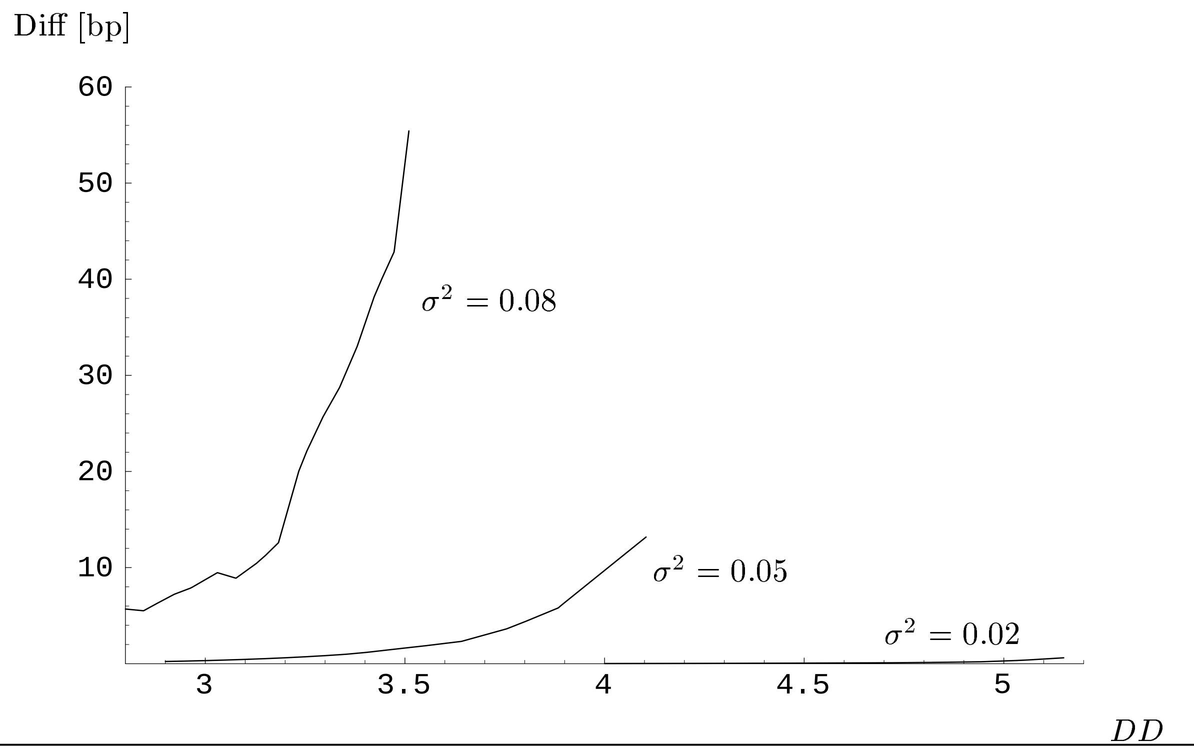
Numerical results: Expected default frequencies

- Expected default frequency and frequency of a recapitalization with subsequent default:
- For large DD, the remaining default risk is only due to corporate leverage changes.



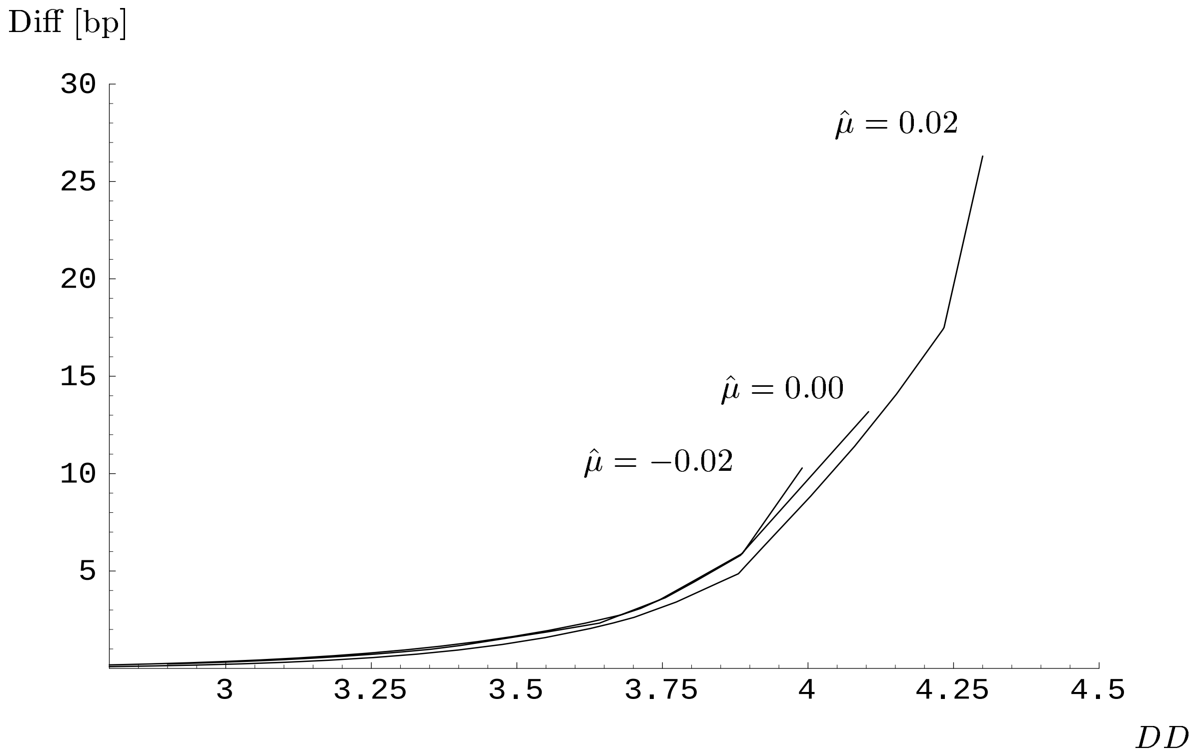
DD and Expected default frequencies

- The graph below displays the understimation of expected default probabilities for various risk levels.



DD and Expected default frequencies

- The graph below displays the understimation of expected default probabilities for various expected cash flow growth rates.



Conclusions (1)

- Capital structure dynamics have important effect on credit risk.
- Traditional DD measure is not a sufficient statistic for true credit risk.
- Mapping from DD into EDF is u-shaped.
- Capital structure dynamics may explain the slow convergence of empirical EDF's to zero.

Conclusions (2)

- Estimating the empirical relationship between DD and expected default frequencies requires conditioning on:
  - firms' volatilities,
  - expected growth,
  - restrictions in existing bond indentures
  - firms' effective corporate tax rates.

Possible extensions

- Consideration of alternative bankruptcy criteria
- Empirical tests
- Allow for multiple debt issues
- Modelling other motives for optimal capital structure (agency considerations etc.)