INTEGRATING MARKET AND CREDIT RISK

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- Overview
  - developments in the risk industry
- Risk measures
  - Quantiles, conditional mean excess value
  - Time horizons
- Distributional assumptions
  - Fat tails and large events
  - Dependencies

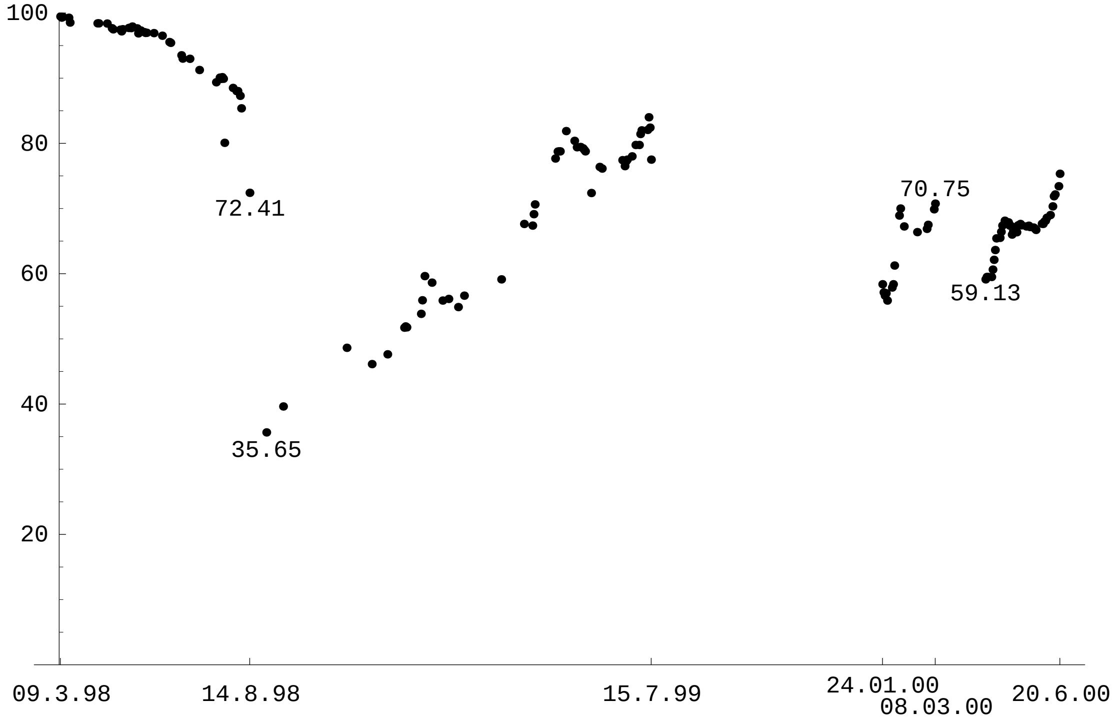
VAR:

- most successful in market risk measurement
- application of VAR like techniques to credit risk

Impact on credit risk:

- Portfolio view:
  Risk of position vs. contribution of position to risk of portfolio
- Continuous valuation
  Default probability demands discounting subject to
  - changes of default probability
  - change of valuation of default risk

  Example: USD bond of the Ukrainian government



Note: Data include default (March 2000)
Largest loss at different date (August 1998)

Backreaction on market risk:

Aspects more clearly seen than before

- Liquidity issues
- Fat tails and event risk
  - Traditional view:
    - Loss probability decreases rapidly beyond chosen percentile
    - Statistical dependencies expressible via correlations
  - In practice: Large events and nonlinear dependencies

  ⇒ Estimation of quantile more difficile

  ⇒ Losses probably much larger than VAR

On both sides:

Refinement of methodology
⇒ integrated view on market and credit risk

E.g.:

- Portfolio view on credit risk
  ⇒ inclusion of market risk as next logical step
- Covering of
  - large events
  - different time horizons

  by market risk methodology

  ⇒ ability to include credit risk as by product

Benefits:

- Derivative products:
  Loss in case of defaults depends on market state at time of default
- Bonds of risky issuers, credit derivatives:
  Market for credit risk

Problems with risk management of these products may turn out to be artifacts of separation between market and credit risk.

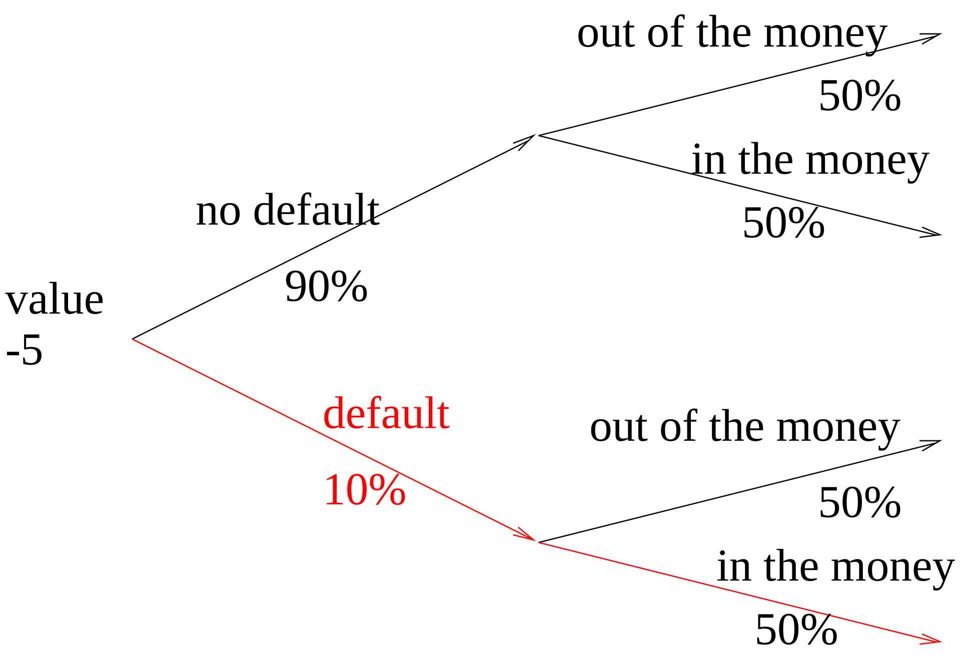
Example: Credit risk of derivatives

Consider two binary options at the money (nominal 100), same underlying, same strike; Probability of exercise 50%

Counter party of short option has no default risk; Counter party of long option has default probability of 10%

A) Traditional view: Calculate market and credit risk separately
   - Market risk: None
   - Credit risk:
     A1) Average exposure (c.f. CreditMetrics) 50
     A2) Maximum likely exposure (more cautious?) 100

B) Integrated calculation of market and credit risk



probability	payoff	P/L
45%	0	+5
45%	0	+5
5%	0	+5
5%	-100	-95

Comparison of results:

A1) Loss of 50 with probability of 10%
A2) Loss of 100 with probability of 10%
B) Loss of 100 with probability of 5%

Remark:

We did not consider

- Statistical dependencies between evolution of market and default probability
- diversification between market and credit risk

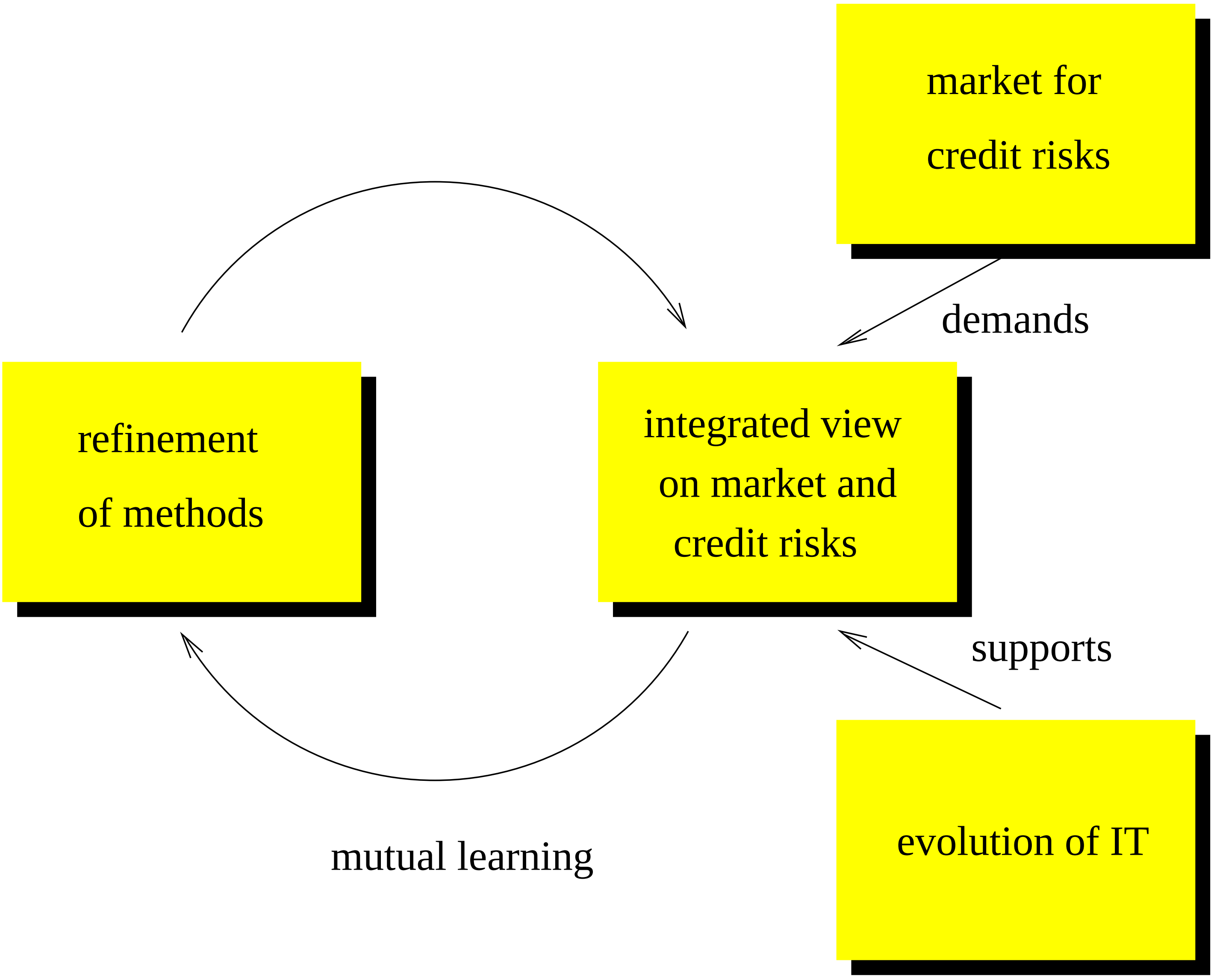
Still accurate measure of risk for derivative product needs integrated view on market and credit risk factors.

Evolution of IT:

- Price of computer power decreased significantly in recent years

$$\Downarrow$$

- Ability to solve problems with numerical methods (e.g. simulations) increased
- Drawback:
  Insight into subtle interplay between risk factors may be lost
- However: Refinement of methodology


market for
credit risks
demands
refinement
of methods
integrated view
on market and
credit risks
supports
mutual learning
evolution of IT

Classical VAR

- Quantile of P/L distribution for confidence level q and time horizon t
- Significant feature: A single positive real number
  - The basic question: Is portfolio acceptable?
    $\Rightarrow$ binary decision, risk $\in \{0, 1\}$
  - However: Smoothness; Risk $\leftrightarrow$ capital
- Drawbacks:
  1. Blind for losses beyond q-quantile
     $\Rightarrow$ sensible only if percentile decreases rapidly beyond q-quantile (e.g. normal distribution)
  2. Not subadditive $\Rightarrow$ not coherent

Remark:

(1) particularly dramatic for credit risks
E.g. single loan, default probability smaller than $1 - q$
However: C.f. short dated out of the money binary option

- Ways out:
  - higher percentile; does not deal with (2)
  - Conditional mean excess value
- Both are
  - possibly difficult to calculate (c.f. below)
  - based on P/L-distribution

Remark: Most of the material presented here applies to any risk measure based on P/L-distribution

Time Horizons

- Market risk – short time horizon (1-10 days)
- Credit risk – long time horizon (~1 year)

Example:

Close swap by swap with different counter party
$\Rightarrow$ closed with respect to market risk
possible loss in case of default

Remark:

loss depends on market state at time of default
$\Rightarrow$ Risk depends on default probability as well as on volatility of market

Solution:

Consider portfolio which is liquidated during evolution towards time horizon

Result for VAR(-like measure) will depend on liquidation strategy

Final result is the minimum of the results for the different liquidation strategies.

An interesting mathematical problem arises: What is the optimal liquidation strategy?

However: This problem need not be solved.
The relevant strategy is the best we can find rather than the theoretically optimal.

Basic distributional assumptions:

Market risk: normal distribution

Applied to

- Distribution of logarithmic returns of market factors
  (E.g. foreign exchange rates, interest rates, equity prices)
- Distribution of P/L of portfolio

Credit risk: binary distribution

Indicates, whether default occurred.

Remarks market risk:

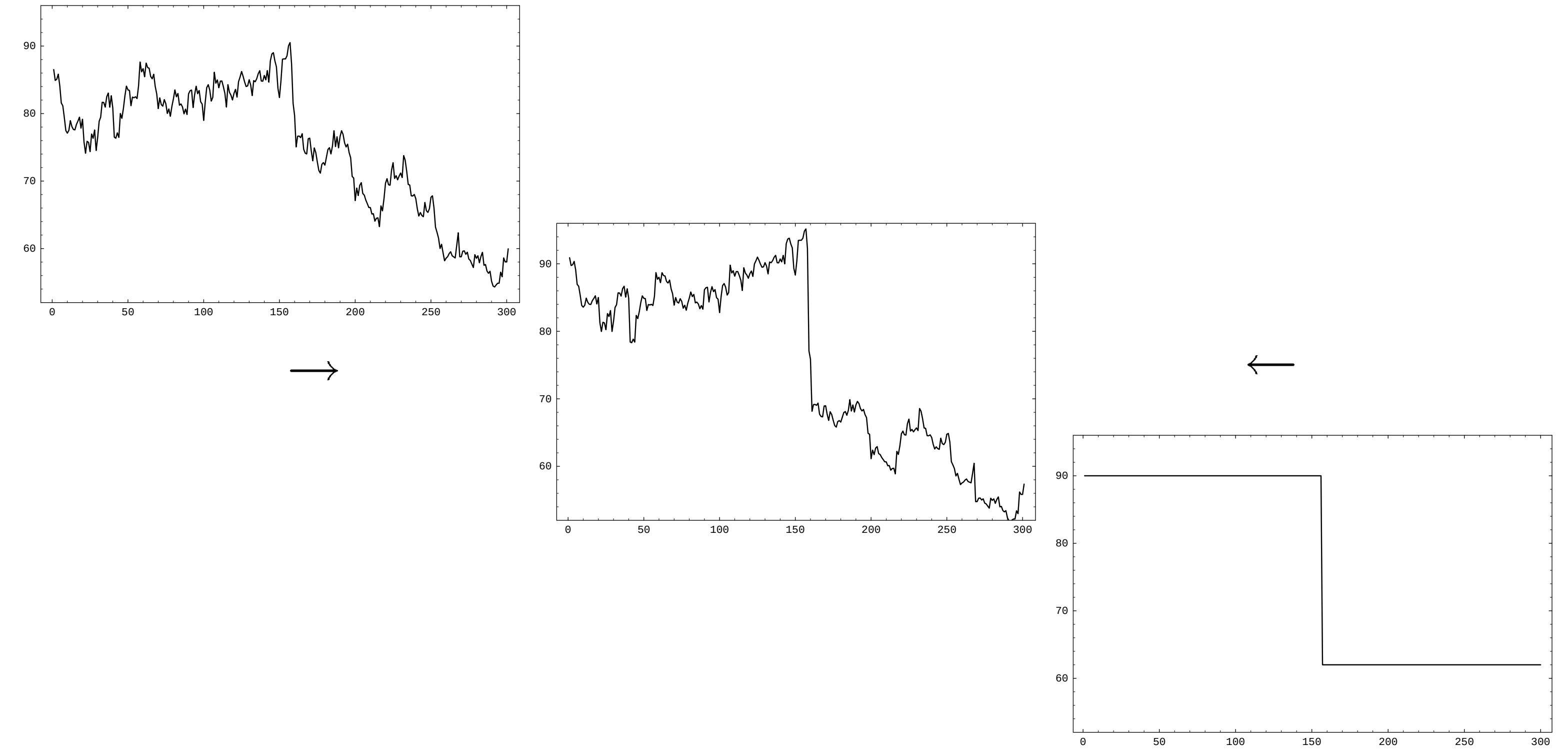
- Empirics: Logarithmic returns of market factors not normally distributed
- Uncertainty of volatility estimate
  Assume $X = \sigma\varepsilon$, ε standard normally distr., σ to be estimated
  Volatility estimate $\hat{\sigma}$ from N returns
  $\Rightarrow \chi^2$-distribution with N degrees of freedom
  $X = \hat{\sigma}\hat{\varepsilon} \Rightarrow \hat{\varepsilon} = \sigma(\varepsilon/\hat{\sigma}) \Rightarrow \hat{\varepsilon}$ StudentT-dist. with N deg. of fr.
- Portfolio
  - Diversification may help
  - Nonlinearity may have opposite effect

Remarks: credit risk

- changes in credit quality and valuation of credit risks
  $\Rightarrow$ continuous fluctuation of value of future cash flow

Conclusion:

Gap between distributional assumptions in market and credit risk is narrowing



To clear up terminology:

Fat tails and large events:

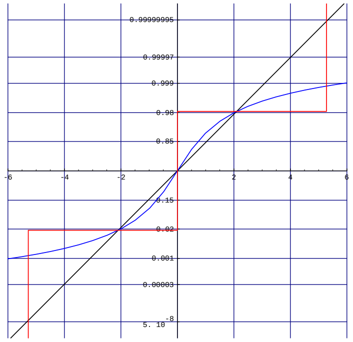
Fat upper tail:

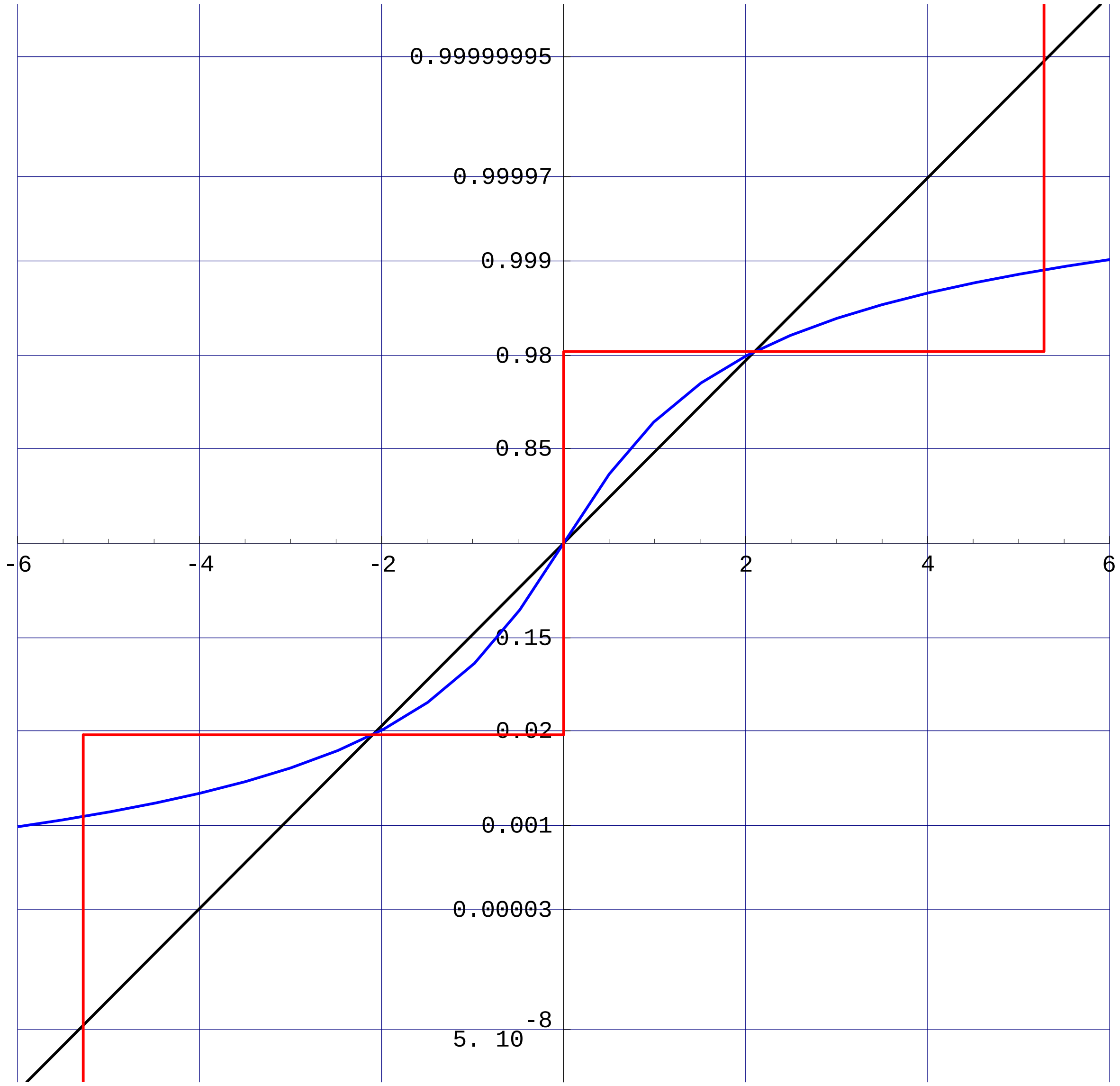
$\text{CDF}(X) < \text{CDF}_{normal}(X)$ for $X > X_0$

Fat lower tail analogously

Consider:

- Normal distribution (black)
- Fat tailed distribution (blue)
- Binary distribution (red)





normal
fat tailed
binary

Remarks:

- High confidence level: Normal distribution will underestimate VAR for fat tailed P/L
- For some region of confidence levels: Converse is true
- For some region of confidence levels: Normal distribution will underestimate VAR, though P/L is not fat tailed (event risk)
- Given confidence level q and volatility σ of P/L distribution:
  Tschebysheff inequality (VAR $\leq \sigma/\sqrt{1-q}$)
  $\Rightarrow$ upper bound for the underestimation of VAR
  by ass. of normal distr.
  Normal distr. will maximally underestimate VAR for binary distr.

Conclusions:

Distinguish

- Fat tails (typical for market risk)
- Event risk (typical for credit risk)

However:

- Finite sample
  $\Rightarrow$ Sees some region of confidence levels only
  $\Rightarrow$ May be unable to distinguish between fat tails and events
- To some extent:
  Event risk may be modeled by fat tails and vice versa.

Example:

Empirical studies on logarithmic returns of equities indicate subexponential distribution (e.g. $p(x)dx \sim dx/x^{\alpha+1}$ for $x \to \infty$)

For subexponential distr. the mean of the exponential is infinite:

$$\int_{-\infty}^{\infty} \exp(x)p(x)dx = \infty$$

Logarithmic returns x subexponential in the upper tail
$\Rightarrow$ Mean of next day's equity price $S = S_0 \exp(x)$ infinite

Does not sound plausible.

Presumably subexponential distribution applies to some region of confidence levels only.

Remark:

Logarithmic returns subexponential in the upper tail
$\Rightarrow$ Conditional mean of excess values infinite
for short equity position

Calculation of cond. mean of excess values crucially depends on assumptions on distribution outside of observed sample.

A unifying Mathematical framework

Consider some simple models in credit risk:

- Single loan
  → binary distribution

- Portfolio of independent loans
  → Poisson distribution

- Stochastic exposure, stochastic recovery rate
  → Compound Poisson distribution:

$$E(e^{ikx}) = \exp[\int (e^{ikx} - 1) N\, q(x)\, dx]$$

N	...	average number of events
$q(x)$	...	Distribution of single event (modeling stochastic exposure, stochastic recovery, ...)

Normal distribution revisited:

- Assume size of single event $\sim 1/\sqrt{N}$
- let $N \to \infty$

$\Rightarrow$ Poisson distribution $\to$ Normal distribution

Interpretation:

- Daily price change results from large number of trader transactions
- Assumption of infinite number of independent transactions appears as an approximation

Generalization:

Infinitely divisible distributions:

- Large variety of distributions
- Can formally be written as compound Poisson distributions with infinite number of events: (Levy Kinchine representation) $N\, q(x) \to \mu(x) \Rightarrow$

$$E(e^{ikx}) = \exp[\int (e^{ikx} - 1 - ik \sin(x))\mu(x)\, dx + ibk]$$

Remarks:

- $\mu(x)\, dx$ ... Levy measure (i.e. $[x^2/(1+x^2)]\, \mu(x)\, dx$ is a bounded, positive measure)
- Levy measures are additive: $\mu_{1,2}$ corr. to indep. r.v. $x_{1,2} \Rightarrow \mu_1 + \mu_2$ corr. to $x_1 + x_2$

Examples:

- $\mu(x) = \delta''(x)$ (second derivative of Dirac Delta function) $\Rightarrow$ normal distribution
- $\mu(x) = \delta(x-a) \Rightarrow$ Poisson distribution

Remark: Any r.v. with infintely divisible distribution can be written as (infinte) sum of independent normally and Poisson distributed r.v.s

- $\mu(x) = P\,\Theta(x)/x^{1+\alpha} + Q\,\Theta(x)/|x|^{1+\alpha}$, $0 < \alpha < 2$ $\Rightarrow$ α-stable distributions

Remarks on α-stable distributions:

- finite variance: normal distribution is the only stable distribution
- infinte variance: 4 parameter family of distributions
  - critical exponent (tail index) α
  - translational parameter (note: mean may be infinte)
  - scaling parameter
  - asymmetry parameter
- multivariate generalisation exists
- possibly useful to describe
  - credit risks
  - market risk in instable markets
  - in some range of quantiles

Dependencies:

Correlation:

- Most popular measure for dependencies among market risk practitioners
- Linear transformation gives uncorrelated variables

Remark:

- Joint normal distribution determined by pairwise correlations
- In general: Joint distribution of $n > 2$ r.v.s not determined by joint distribution of each pair.

Empirics:

Nonlinear statistical dependencies seen in large events

Determining these dependencies	$\leftrightarrow$	Estimation of joint probability of default

In particular:

- Sparse data in the estimation of nonlinear dependencies of market data
- This is typical problem in credit risk management.
- The process of mutual learning will be most fruitful here.

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