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CDO Risk and Valuation

Darrell Duffie and Nicolae Gârleanu

Stanford University

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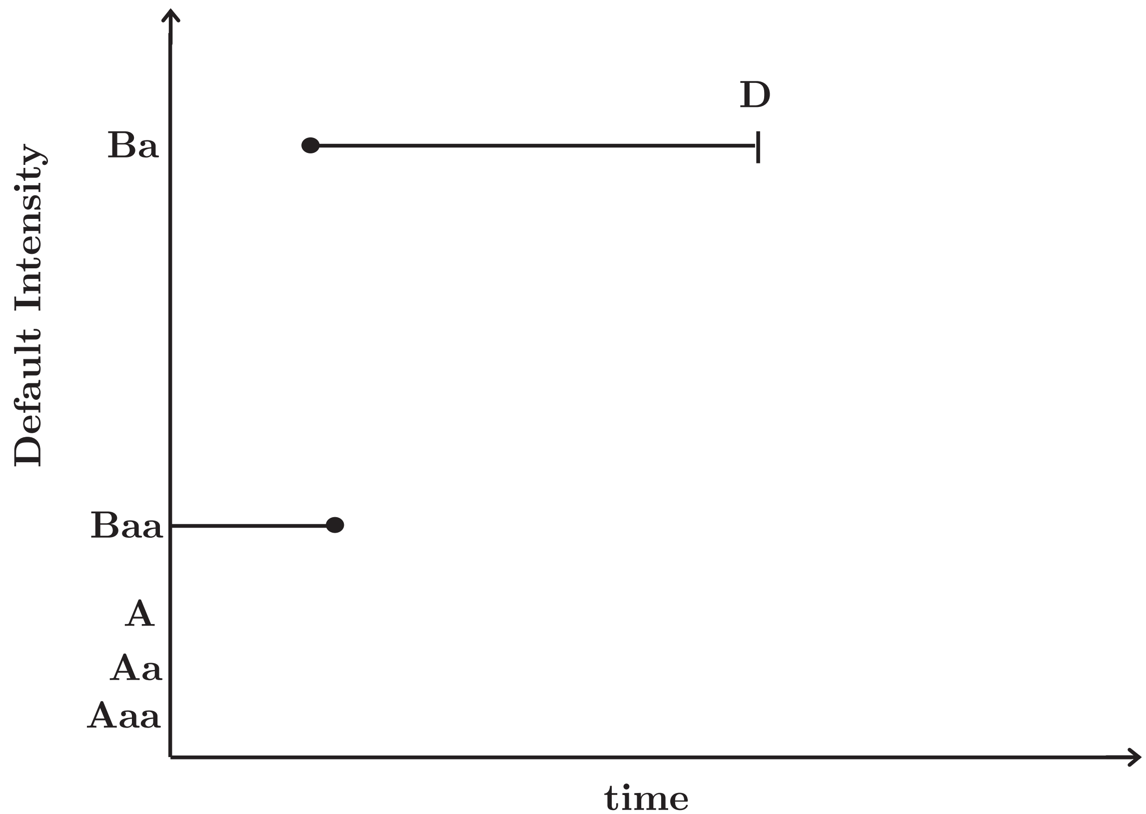
Alternative Approaches to Correlation

- CreditMetrics: ratings transitions with correlation.
- KMV: Joint EDFs with correlated distances to default.
- Correlated default intensity processes.
- Joint credit events.

Intensity Paths (Rating-Based Model)



Average One-Year Transition Rates by Rating Category

	–Rating at year end (%)–							
Initial rating	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**
AAA	89.48	7.26	0.47	0.08	0.04	0.00	0.00	0.00
AA	0.62	89.99	6.55	0.58	0.06	0.11	0.03	0.00
A	0.07	2.18	87.95	4.91	0.54	0.24	0.01	0.04
BBB	0.04	0.25	5.23	82.66	4.54	0.96	0.16	0.22
BB	0.04	0.09	0.55	7.04	73.98	7.17	1.01	0.92
B	0.00	0.09	0.25	0.41	6.14	73.15	3.50	4.82
CCC	0.16	0.00	0.32	0.64	2.09	10.43	52.01	20.39

Source: Standard and Poor's, March, 1999, Table 9

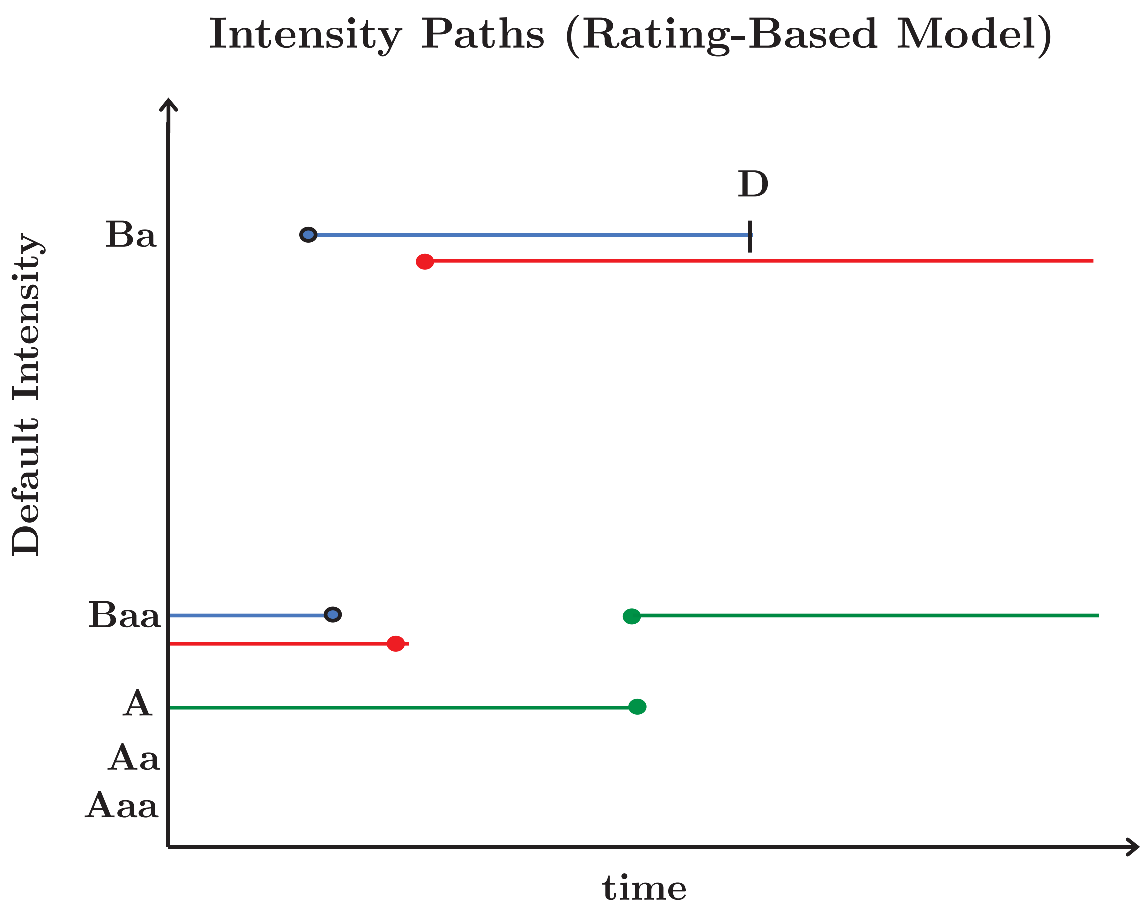


Figure 1: Correlated Ratings-Based Default Intensity for 3 Issuers

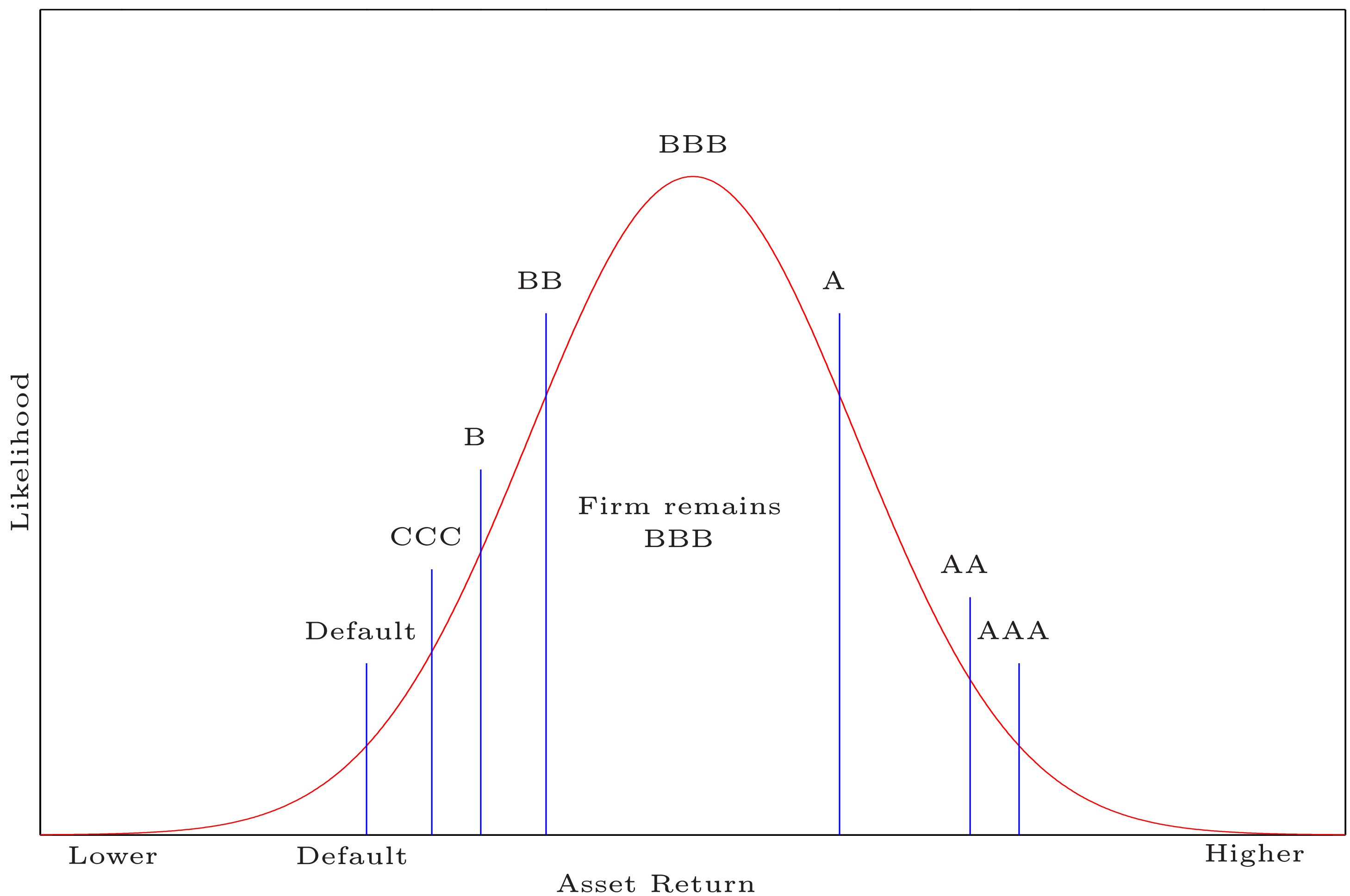


Figure 2: Transition Probabilities of BBB firm from Asset Return Likelihood

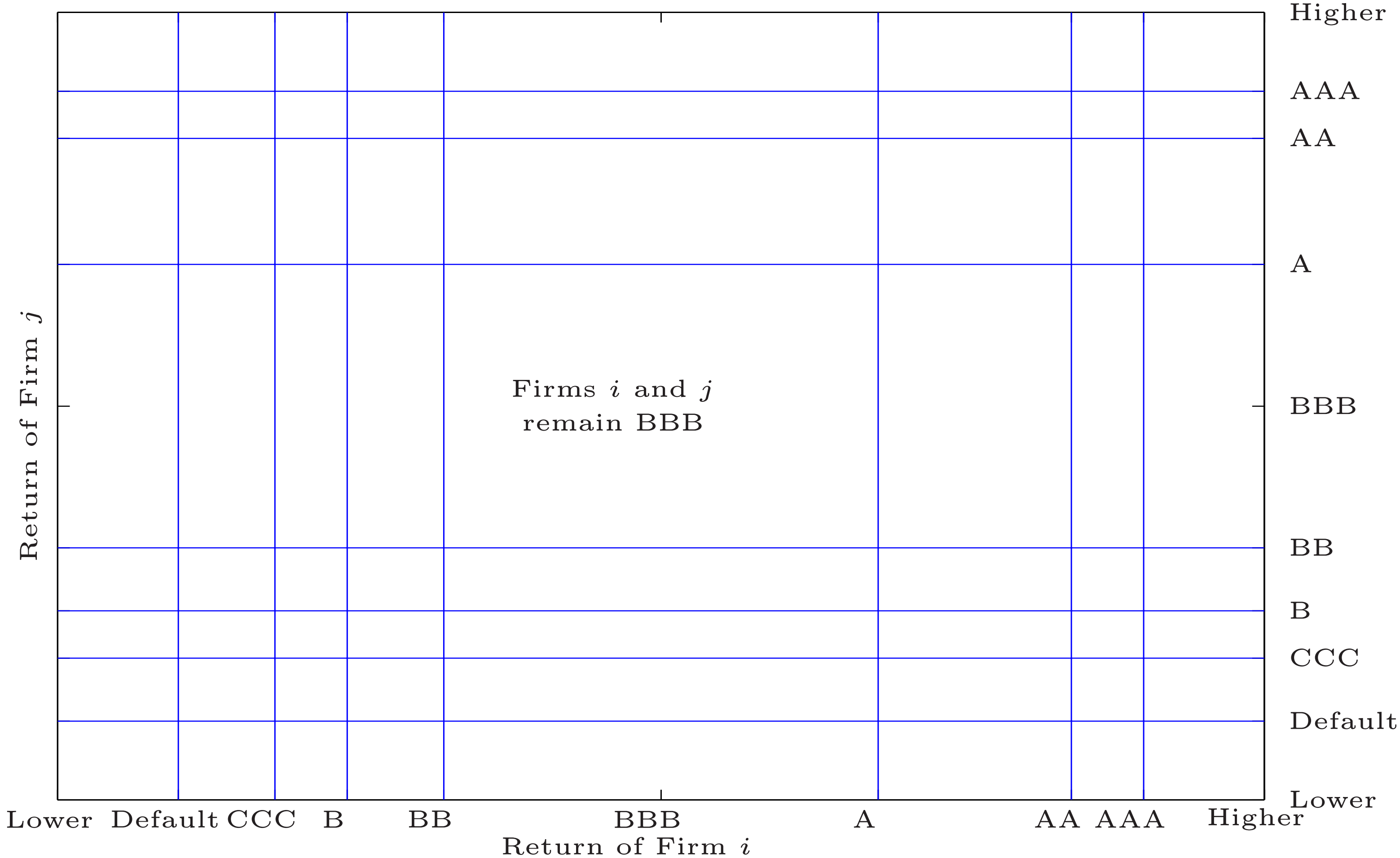


Figure 3: Joint transition regions for two BBB firms

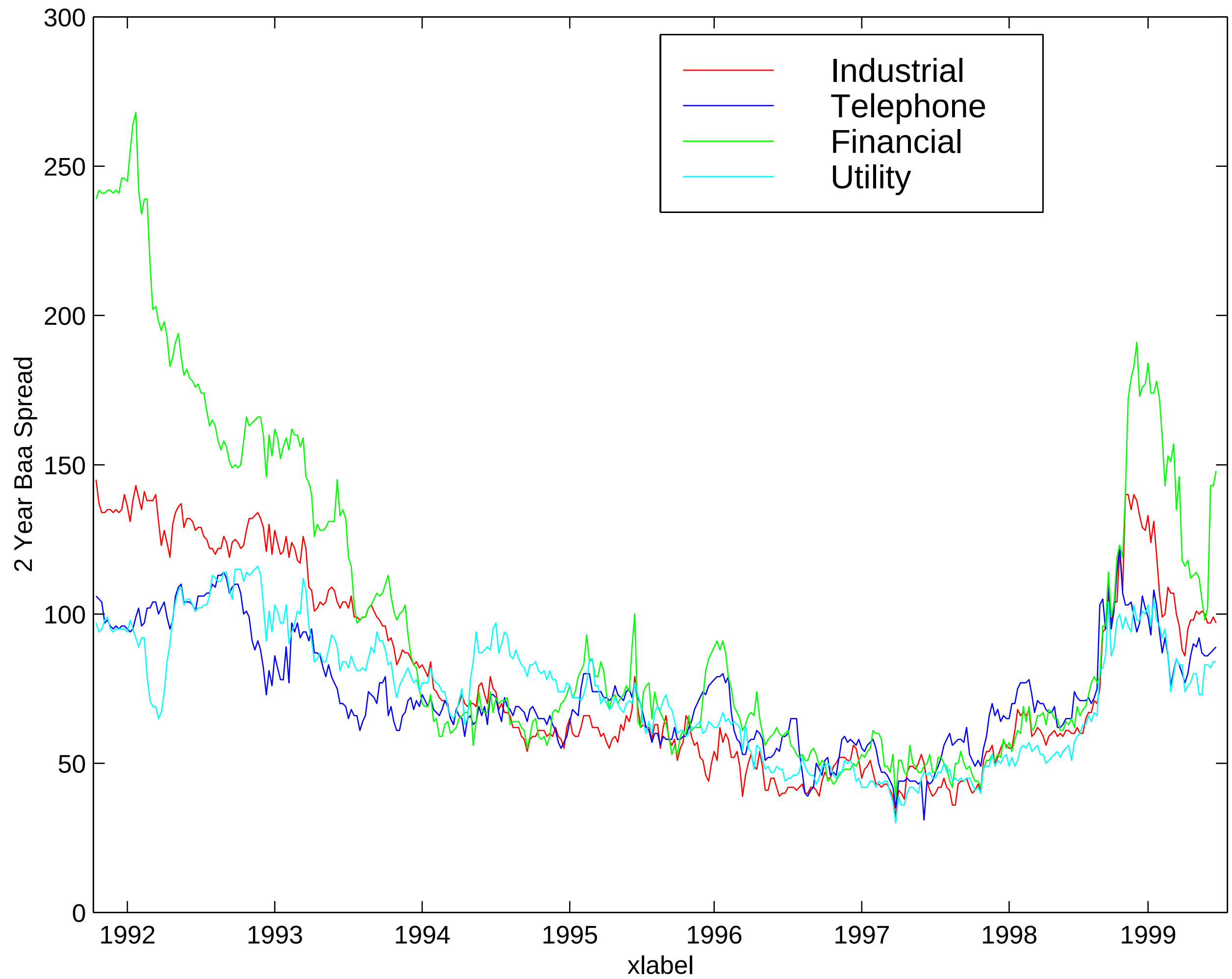


Figure 4: 2-year Baa-rating spread by sectors.

Default Intensities with Correlation

- One could model default intensities $\lambda_1, \ldots, \lambda_n$ in various ways, with correlation.
- By incorporating correlation in the intensities, for example through common jump times, one can build rich and tractable models of default correlation.
- From such models, one can easily simulate correlated defaults and correlated changes in default probabilities over various horizons.

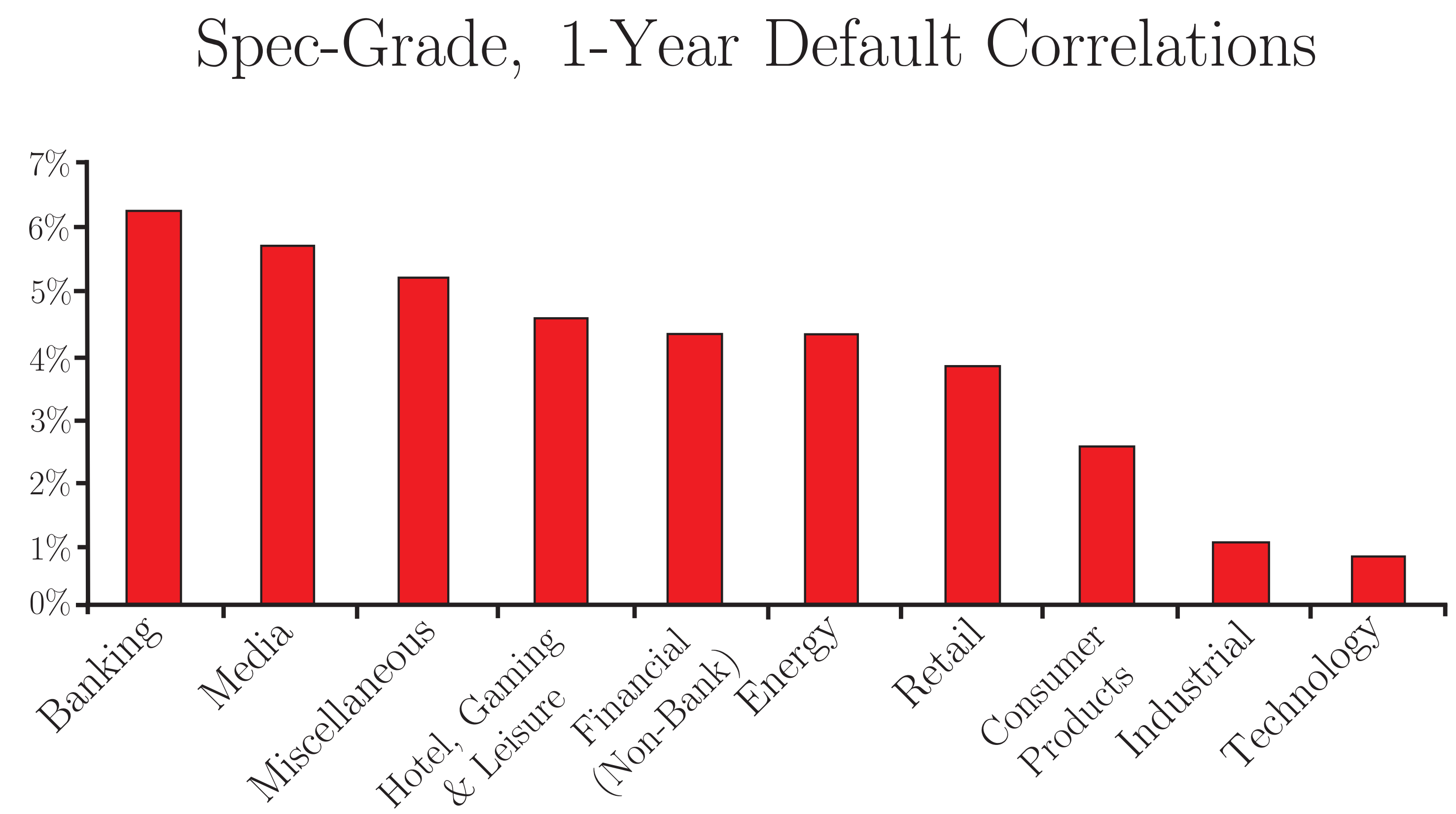


Figure 5: Moody's estimates of default correlations, based on incidence of default, by industry.

Doubly-Stochastic Joint Default Models

With the doubly-stochastic model, conditional on the intensity processes $\lambda_1, \ldots, \lambda_n$ of the n names: *(i)* the respective default times $\tau_1, \ldots, \tau_n$ are independent, and *(ii)* τ_i is the first arrival of a Poisson with deterministic time-varying intensity process λ_i.

This means roughly that there are no credit events that could simultaneously cause more than one default and that the only source of default-time correlation is through variation in (intensities) conditional default probabilities.

Estimating Correlated Intensity Processes

- One approach is to use the incidence of actual historical default data, and estimate, say, a logit model that predicts default based on covariates. Data on defaulted firms, however, is extremely limited.

- Another approach is to use historical estimates of the conditonal probability p_{it} at time t that entity i survives for one year. KMV and Moodys, for example, have such historical data. Such a database may provide many observations.

- Then, given the available data across all past times (t) and names (i), one can estimate a model for the joint evolution of the default intensity processes $\lambda_1, \ldots, \lambda_n$ that reflects the observed correlated variation over time in $p_{1t}, \ldots, p_{nt}$.

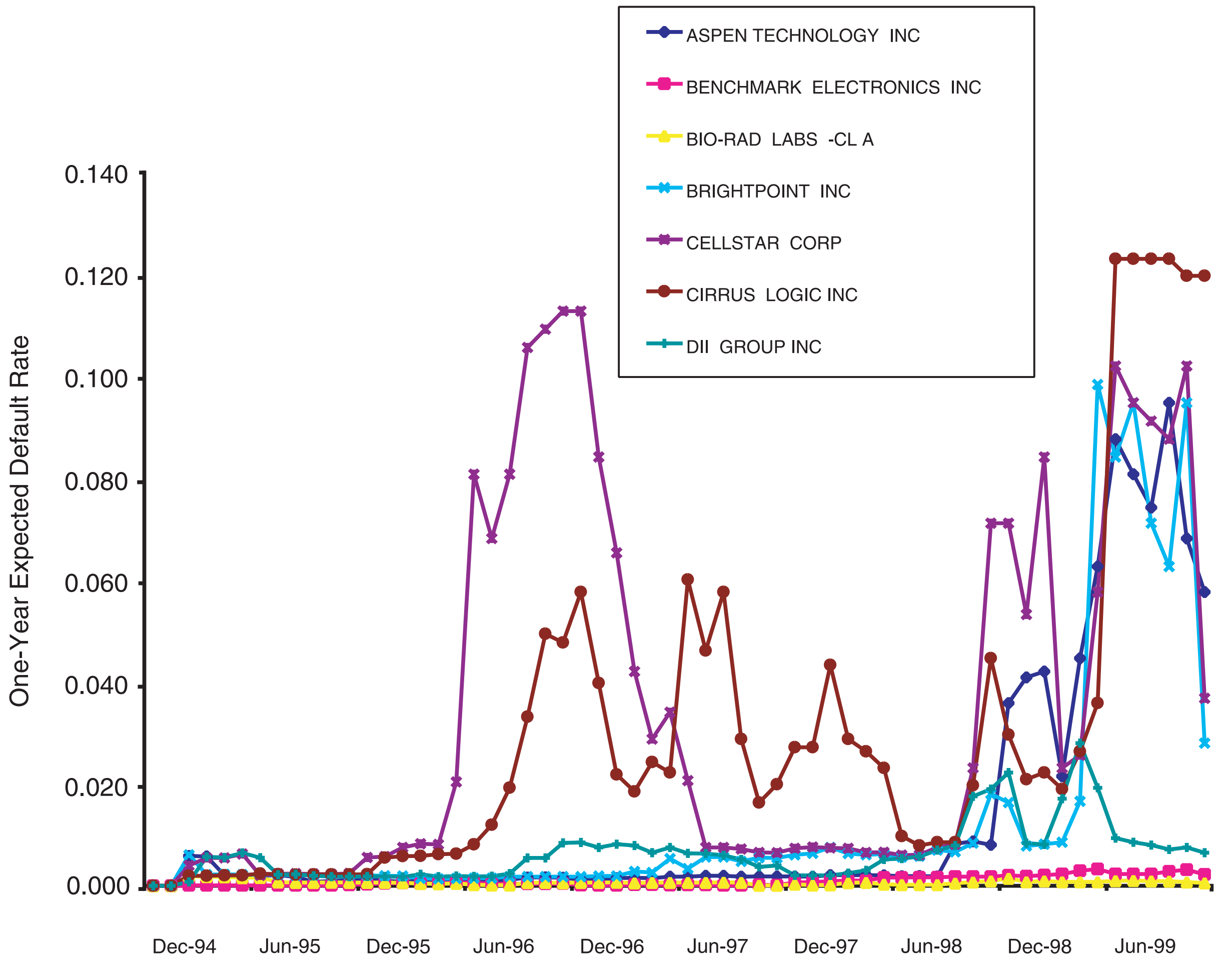


Figure 6: One-Year Expected Default Rate $-\log(p_t)$, Moody's data for p_t, 1-year survival probability p_t at t, selected electronics firms.

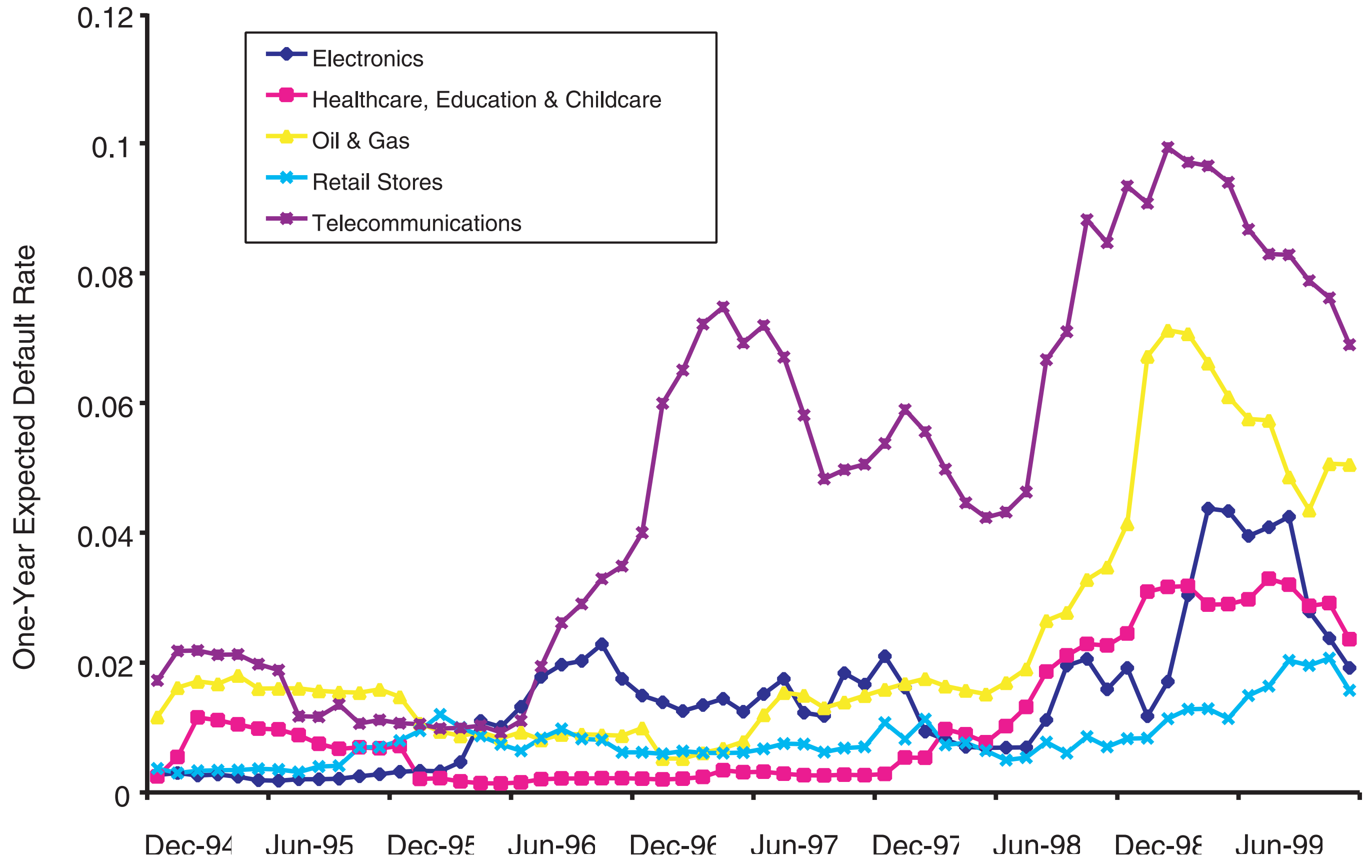


Figure 7: Average of One-Year Expected Default Rate $-\log(p_t)$, Moody's data for the survival prob p_t, averages of 10 selected firms within each of 5 selected industries.

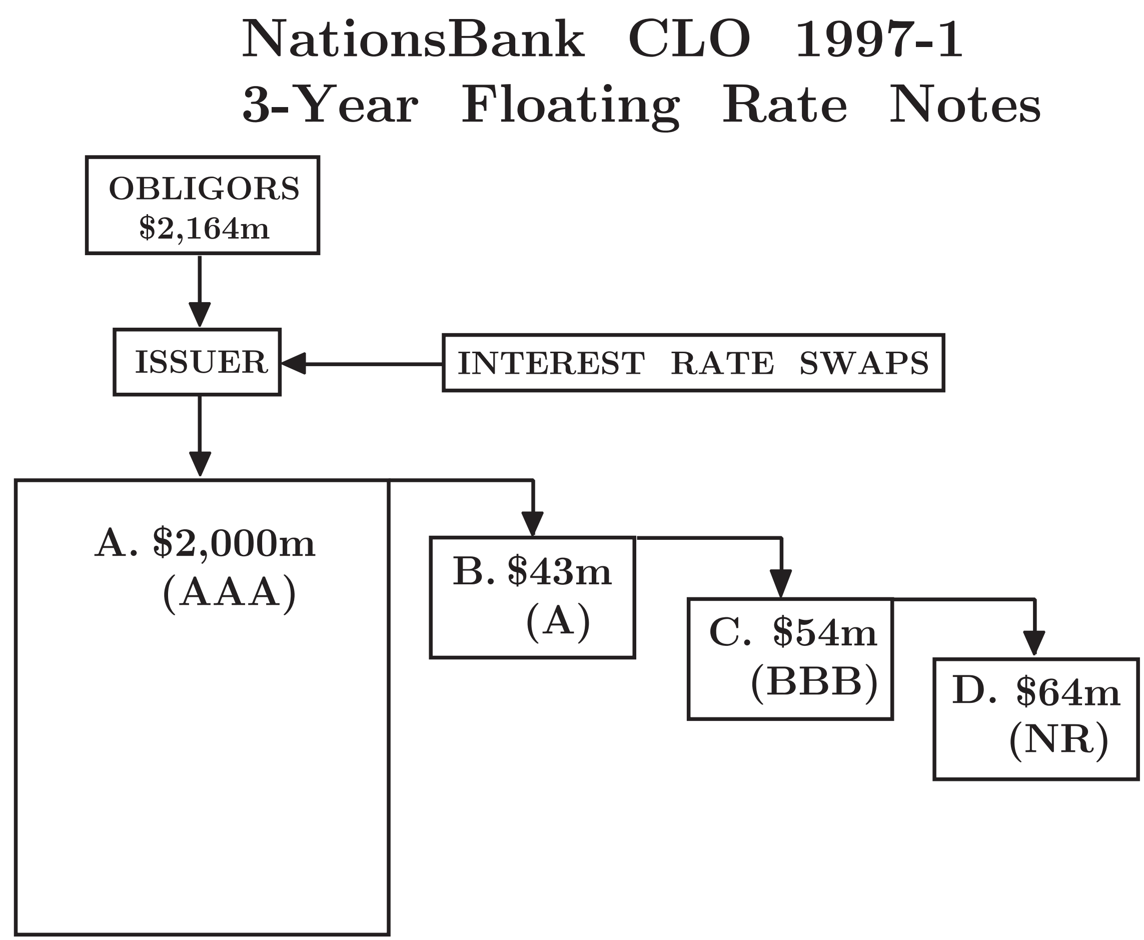
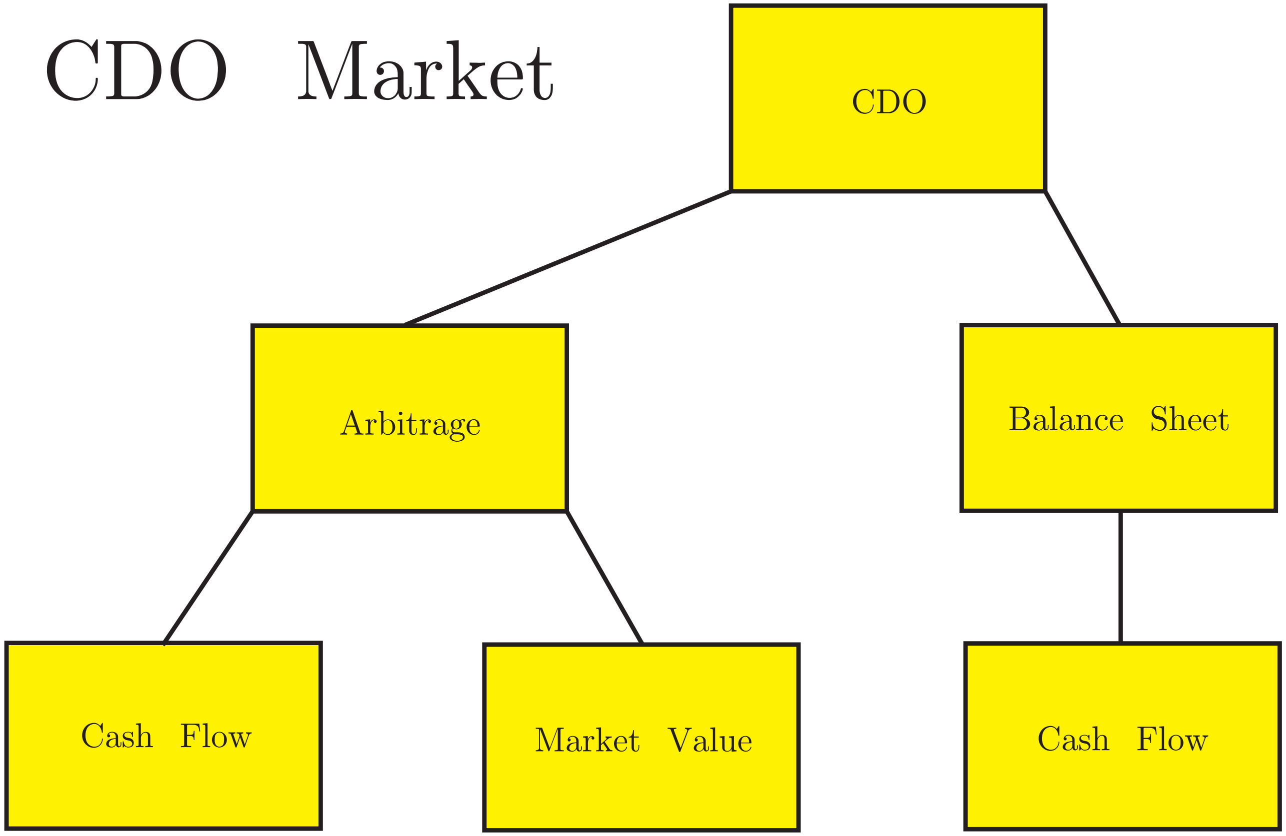


Figure 8: CDO Tranches

CDO Market

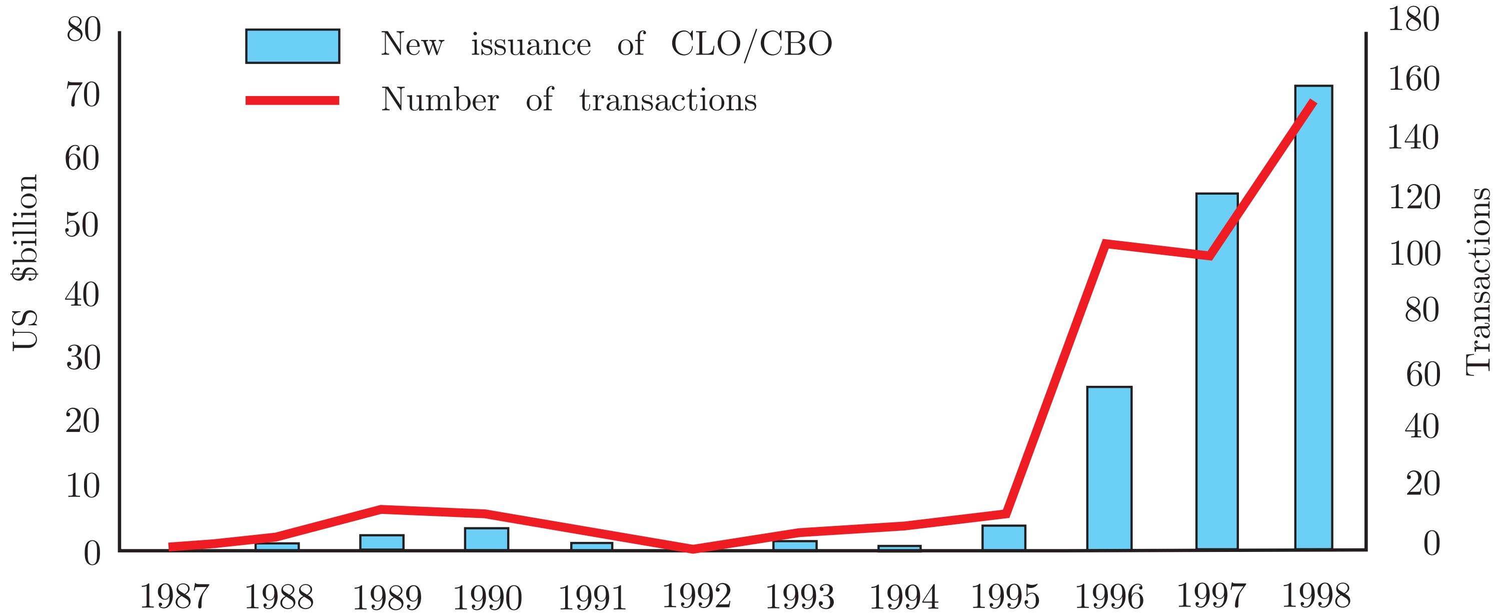


Source: Morgan Stanley

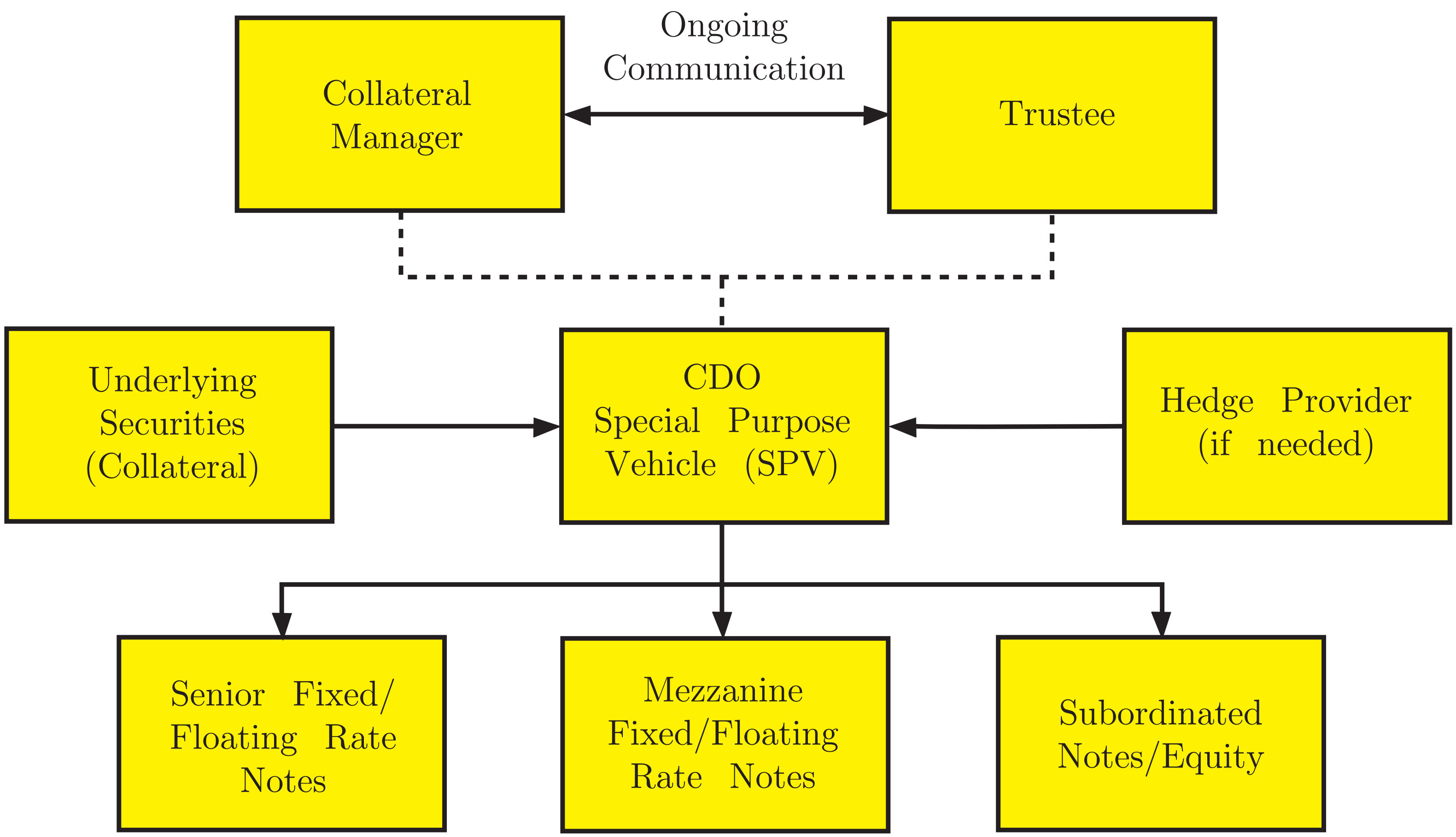
Overview of Collateralized Debt Obligations

- Useful for balance-sheet reductions and liquidity enhancement.
- Tranching is prioritized over defaults to mitigate pricing effects of adverse selection, moral hazard, illiquidity.
- Pricing of subordinated tranches depends heavily on level of default correlations.
- Pricing, risk measurement, and rating technologies are limited by data and by simulation technology.

Global CLO/CBO Issuance: 1987--1998



Source: Asset Backed Alert, Deutsche Bank estimates



Source: Morgan Stanley

Figure 9: Typical CDO Contractual Relationships

Diversity Score

The number n of bonds in an idealized comparison portfolio such that:

1. The total face value of the comparison portfolio is the same as the total face value of the collateral pool.
2. The bonds of the comparison portfolio have equal face values.
3. The comparison bonds are equally likely to default, and their default is independent.
4. The comparison bonds are, in some sense, of the same average default probability as the participations of the collateral pool.
5. The comparison portfolio has, according to some measure of risk, the same total loss risk as does the collateral pool.

Table 1: Moody's Diversity Scores for Firms within an Industry

Number of Firms in Same Industry	Diversity Score
1	1.00
2	1.50
3	2.00
4	2.33
5	2.67
6	3.00
7	3.25
8	3.50
9	3.75
10	4.00
>10	Evaluated on a case-by-case basis

Source: Moody's Investors Service

Table 2: Rating Scores Used to Derive Weighted Average Ratings

	Moody's	Fitch	DCR
Aaa/AAA	1	1	0.001
Aa1/AA+	10	8	0.010
Aa2/AA	20	10	0.030
Aa3/AA-	40	14	0.050
A1/A+	70	18	0.100
A2/A	120	23	0.150
A3/A-	180	36	0.200
Baa1/BBB+	260	48	0.250
Baa2/BBB	360	61	0.350
Baa3/BBB-	610	94	0.500
Ba1/BB+	940	129	0.750
Ba2/BB	1,350	165	1.000
Ba3/BB-	1,780	210	1.250
B1/B+	2,220	260	1.600
B2/B	2,720	308	2.000
B3/B-	3,490	356	2.700
CCC+	NA	463	NA
Caa/CCC	6,500	603	3.750
CCC-	NA	782	NA
<Ca/<CCC-	10,000	1,555	NA

Source: Morgan Stanley, Moody's Investors Service, Fitch Investors Service, and Duff and Phelps Credit Rating

Table 3: Basic Features of Calhoun CBO, Ltd. (Source: Morgan Stanley)

- Manager: American Express Asset Management
- 82.2% U.S. high yield bonds, 17.8% emerging market bonds
- Fixed rate: 91.4%
- Interest rate hedge: $1880 mm notional
- Minimum average rating: B2. Minimum diversity score: 40.
- Reinvestment period: 5 years. Non-call period: 3 years
- 12-year final maturity.
- Aa2 rated Senior Notes, 7.7 yr AL (70.18% of transaction)
- Baa3 rated Second Priority Senior Notes, 11.0 yr AL (15.79%)
- Senior Subordinated Notes (7.02%). Junior Subordinated Notes (7.02%)

Numerical Examples

- Collateral: 100 ten-year straight coupon bonds.
- Default Modeling: jump-diffusion intensities with correlation.
- Recovery: independent and uniform $[0, 100]$.
- 3 tranches: senior bond, mezzanine bond, junio residual.
- Simple prioritization schemes: uniform and fast.

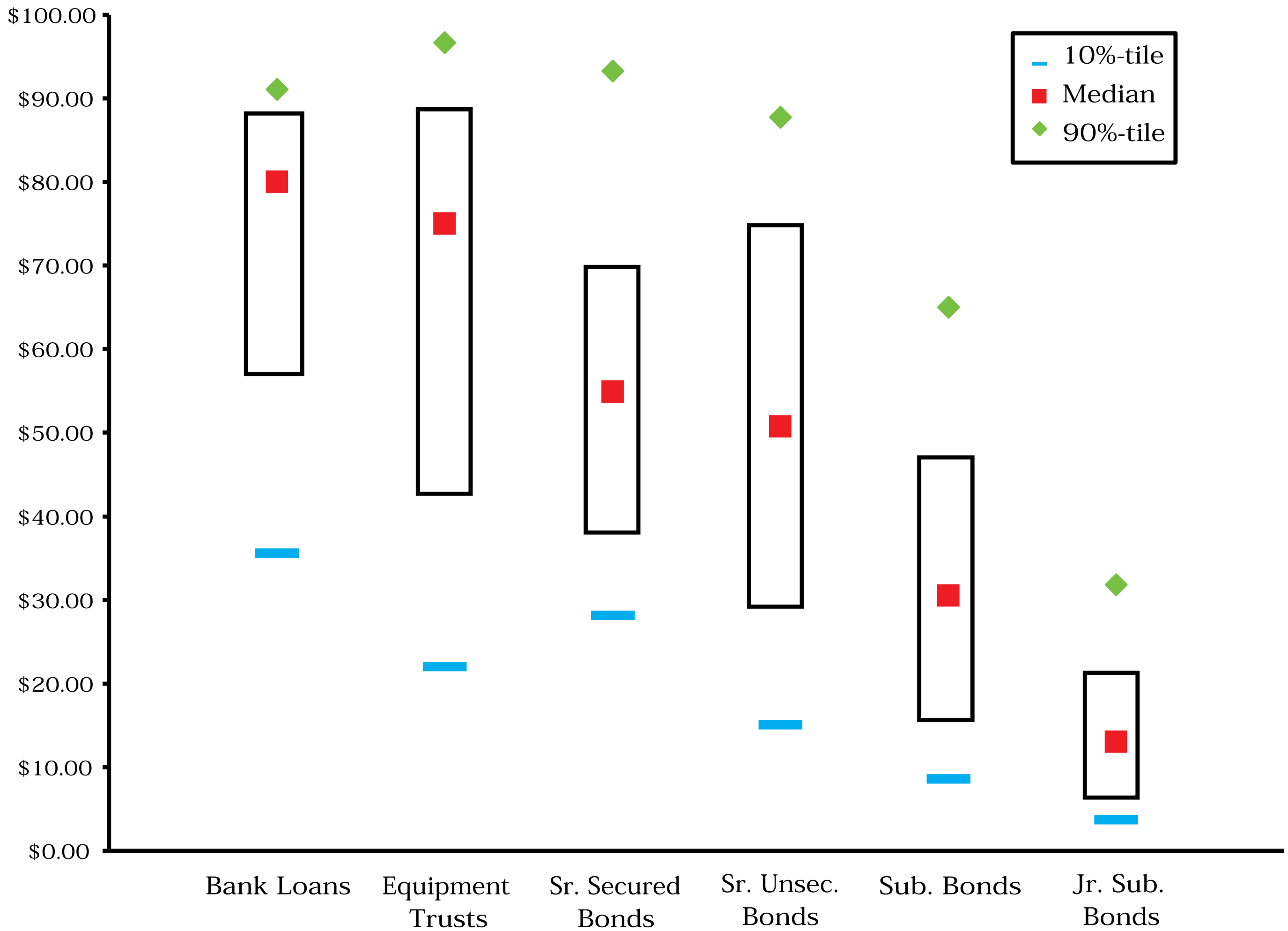


Figure 10: Default Recovery Distributions (Empirical)

Intensity Model

- Default intensities $\lambda_i, \ldots, \lambda_{100}$.
- $\lambda_i = X_c + X_i$, where X_c and X_i are independent "basic affine processes:"

$$\begin{aligned} dX_i(t) &= \kappa(\theta_i - X_i(t))dt + \sigma\sqrt{X_i(t)}\,dW_i(t) + \Delta J_i(t) \\ dX_c(t) &= \kappa(\theta_c - X_c(t))dt + \sigma\sqrt{X_c(t)}\,dW_c(t) + \Delta J_c(t) \end{aligned}$$

- J_i jumps independently at mean rate l_i.
- J_c jumps independently at mean rate l_c.
- All jump sizes are independent exponentials with mean μ.

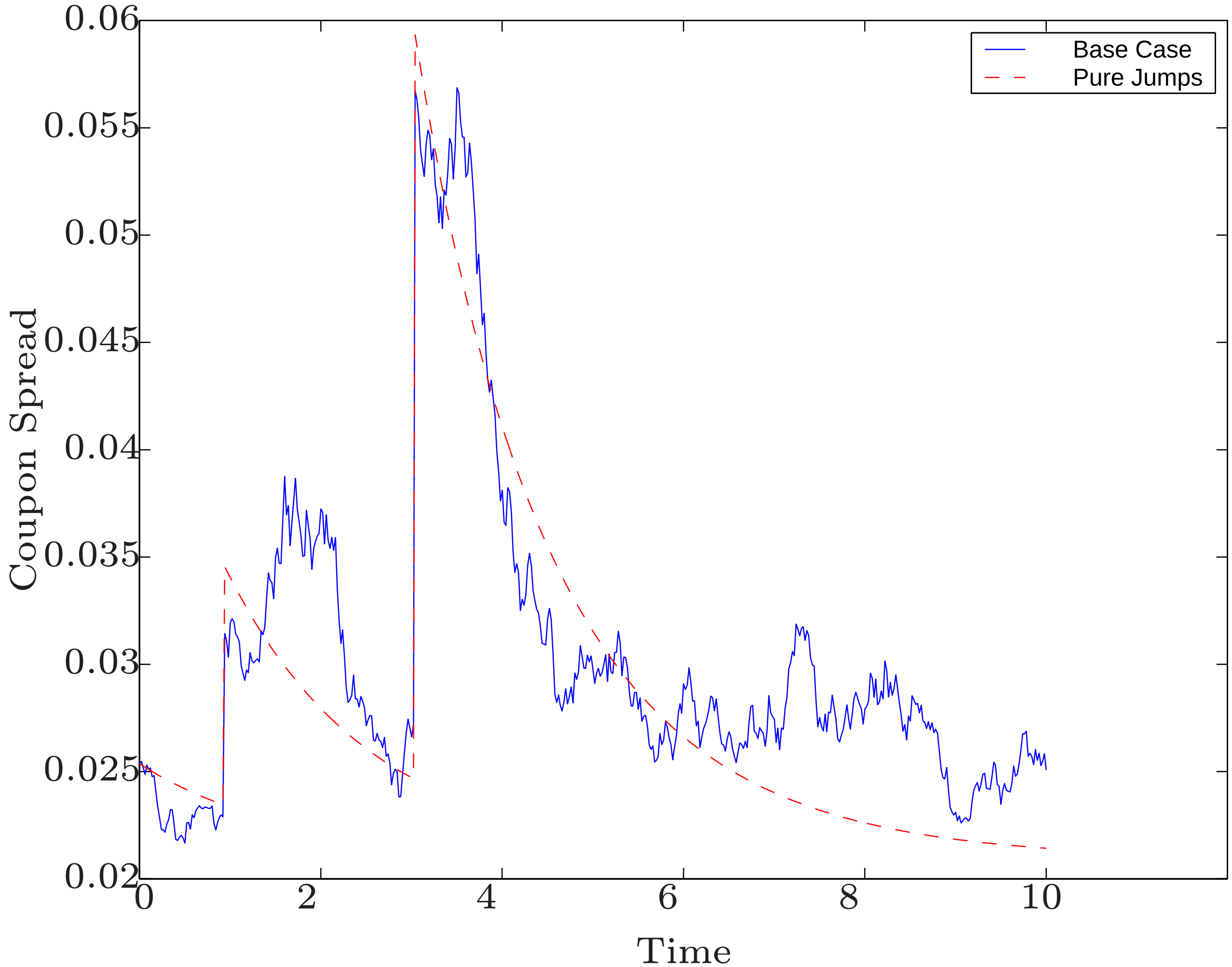


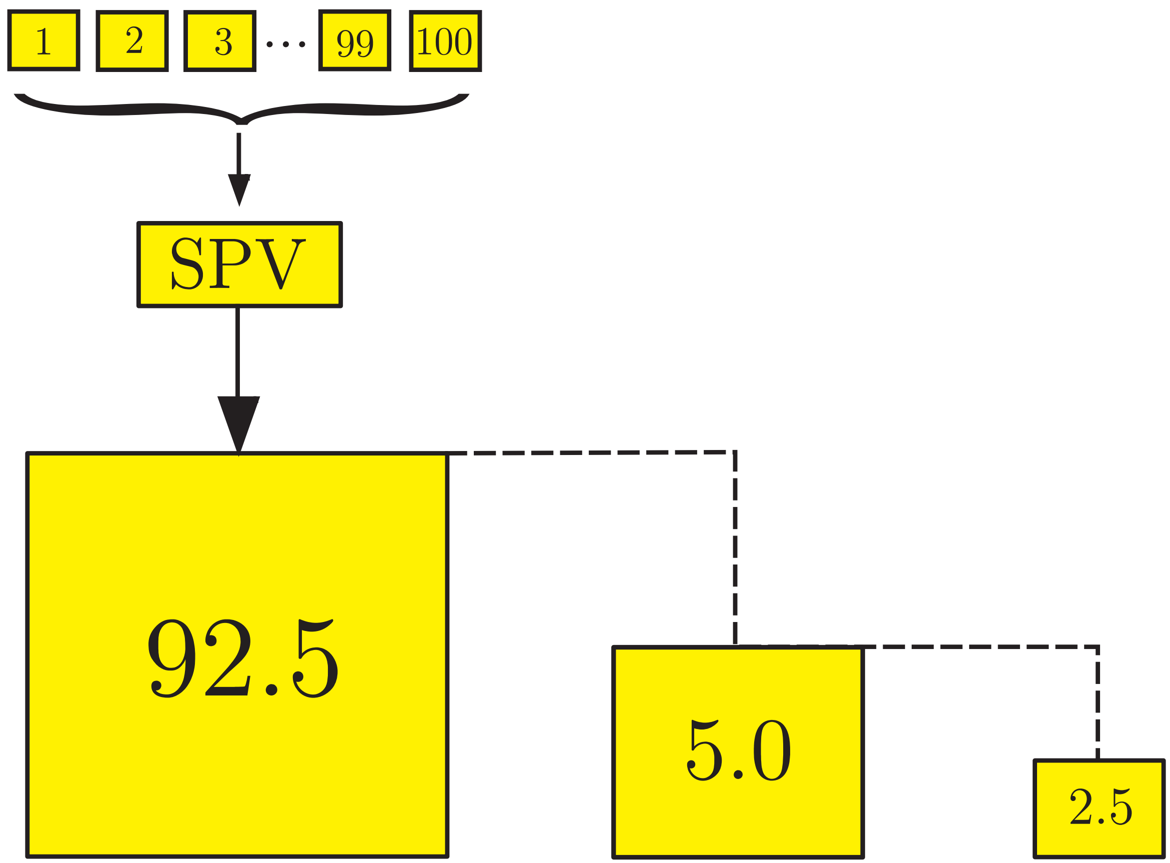
Figure 11: New 10-year par-coupon spreads for the base-case parameters and for the pure-jump intensity parameters.

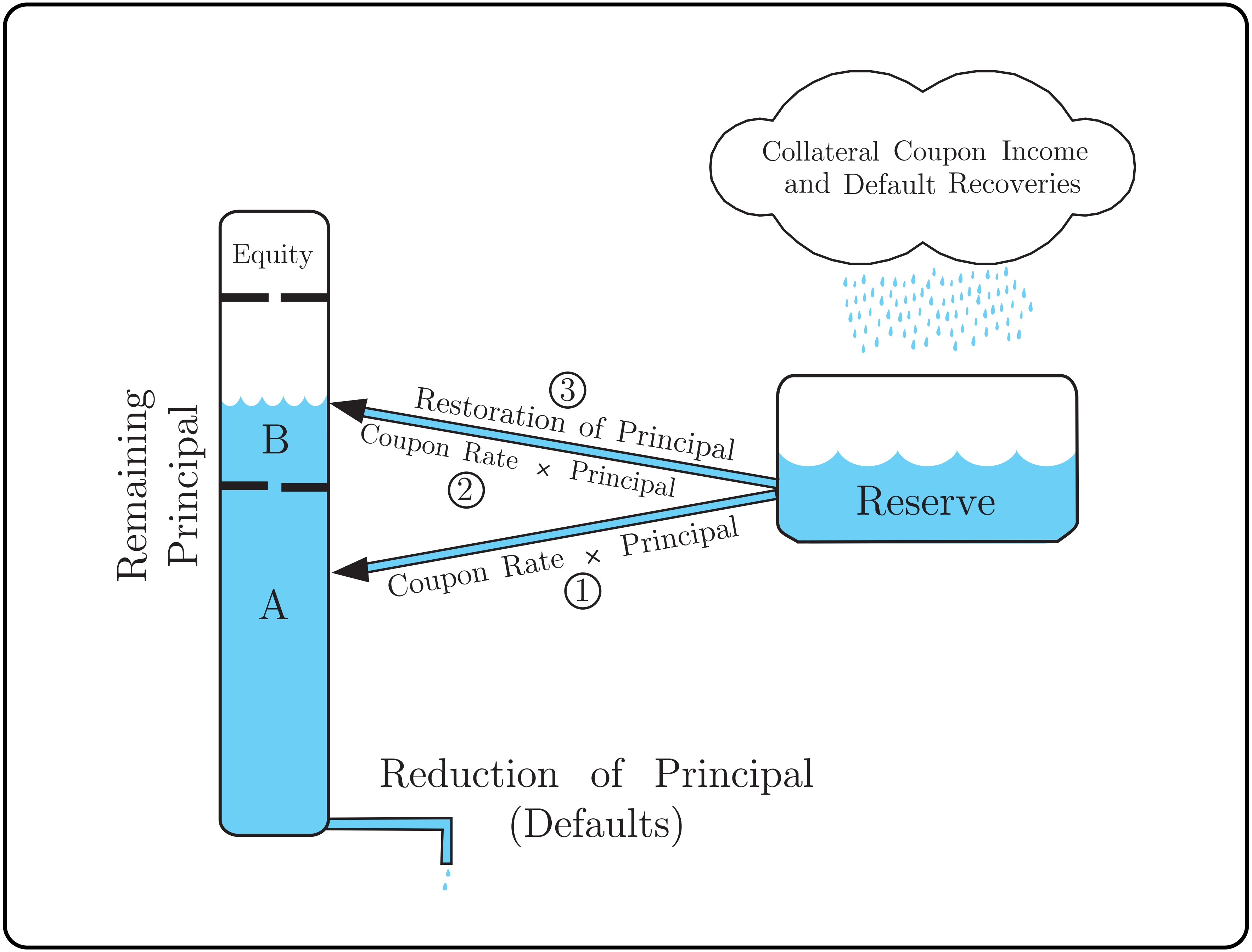
Proposition. Suppose X and Y are independent basic affine processes with the same mean reversion κ and vol σ. Then $X + Y$ is a *one-factor* basic affine process.

Risk-Neutral Base-Case Intensity Parameters

- $\kappa = 0.6$ (mean reversion of 60 percent per year).
- $\theta = \theta_i + \theta_c = \sigma^2 = 0.02$
- $l = l_i + l_c = 0.2$ (mean rate 2 jumps per 10 years)
- $\mu = 0.1$ (mean jump size of intensities of 1000 basis points)
- $\lambda_i(0) = \theta + \mu l/\kappa = 533$ basis points.
- $\frac{\theta_c}{\theta} = \frac{l_c}{l} = \rho = 0.5$ (initial correlation)

CBO Example




Collateral Coupon Income
and Default Recoveries
Equity
B
A
Remaining
Principal
③ Restoration of Principal
Coupon Rate × Principal ②
Coupon Rate × Principal ①
Reserve
Reduction of Principal
(Defaults)

Coupon Rates and Tranches

- Risk-free rate 6%.
- Par coupon spread on collateral approximately 250 basis points.
- A Tranche: Face Value 92.5, par spread of 18 basis points.
- B Tranche: Face Value 5.0, par spread of 710 basis points.
- Equity residual: Base-case value of 2.5.

Table 4: Risk-Neutral Default Parameter Sets

Set	κ	θ	σ	ℓ	μ	Spread	var_{∞}
1	0.6	0.0200	0.141	0.2000	0.1000	254 bp	0.42%
2	0.6	0.0156	0.000	0.2000	0.1132	254 bp	0.43%
3	0.6	0.0373	0.141	0.0384	0.2500	253 bp	0.49%
4	0.6	0.0005	0.141	0.5280	0.0600	254 bp	0.41%

Table 5: Conditional probabilities of default and diversity scores

Set	$P(d_i)$	$\rho = 0.1$		$\rho = 0.5$		$\rho = 0.9$	
		$P(d_i \mid d_j)$	divers.	$P(d_i \mid d_j)$	divers.	$P(d_i \mid d_j)$	divers.
1	0.386	0.393	58.5	0.420	21.8	0.449	13.2
2	0.386	0.393	59.1	0.420	22.2	0.447	13.5
3	0.386	0.392	63.3	0.414	25.2	0.437	15.8
4	0.386	0.393	56.7	0.423	20.5	0.454	12.4

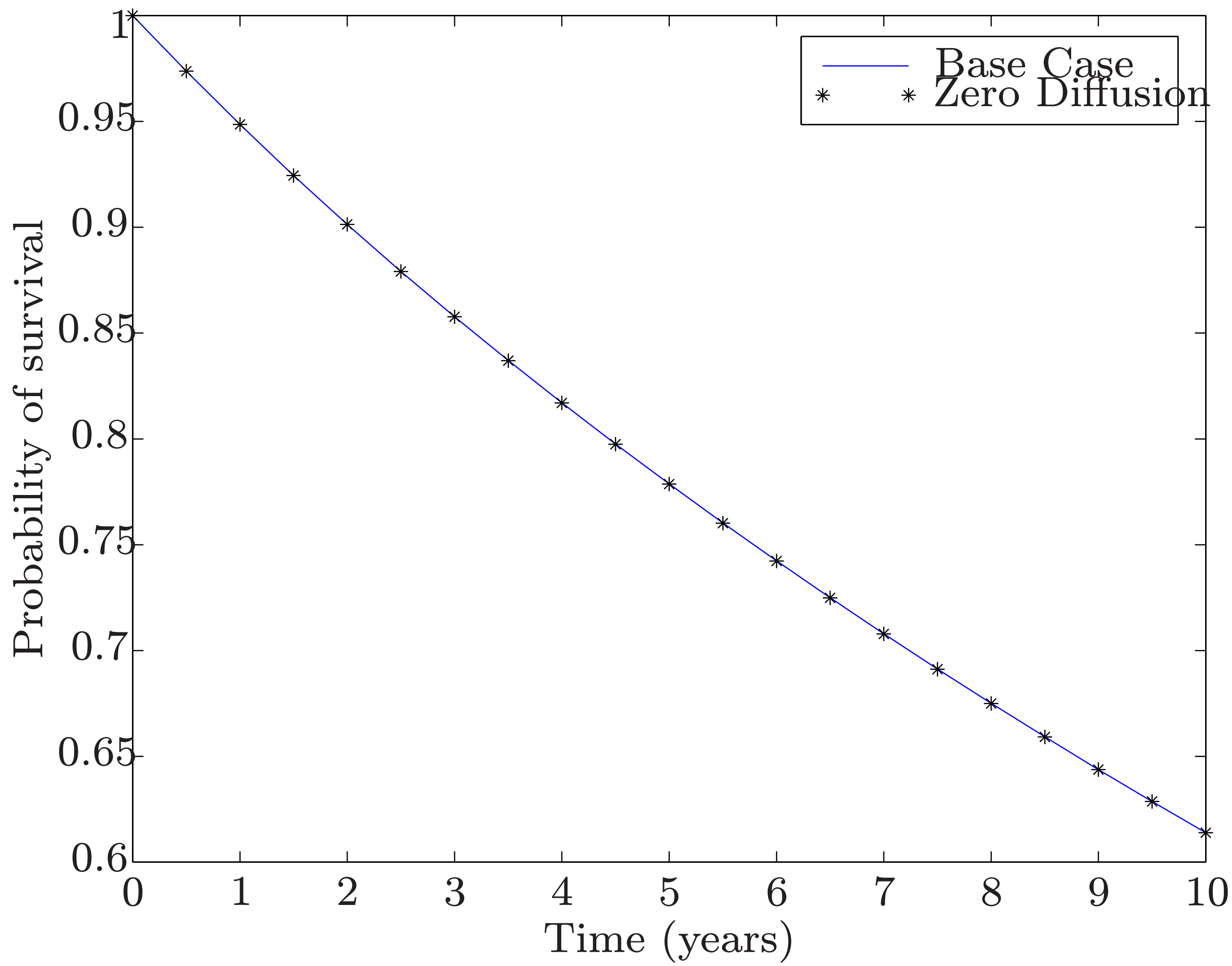


Figure 12: Term Structures of Survival Probabilities, With and Without Diffusion

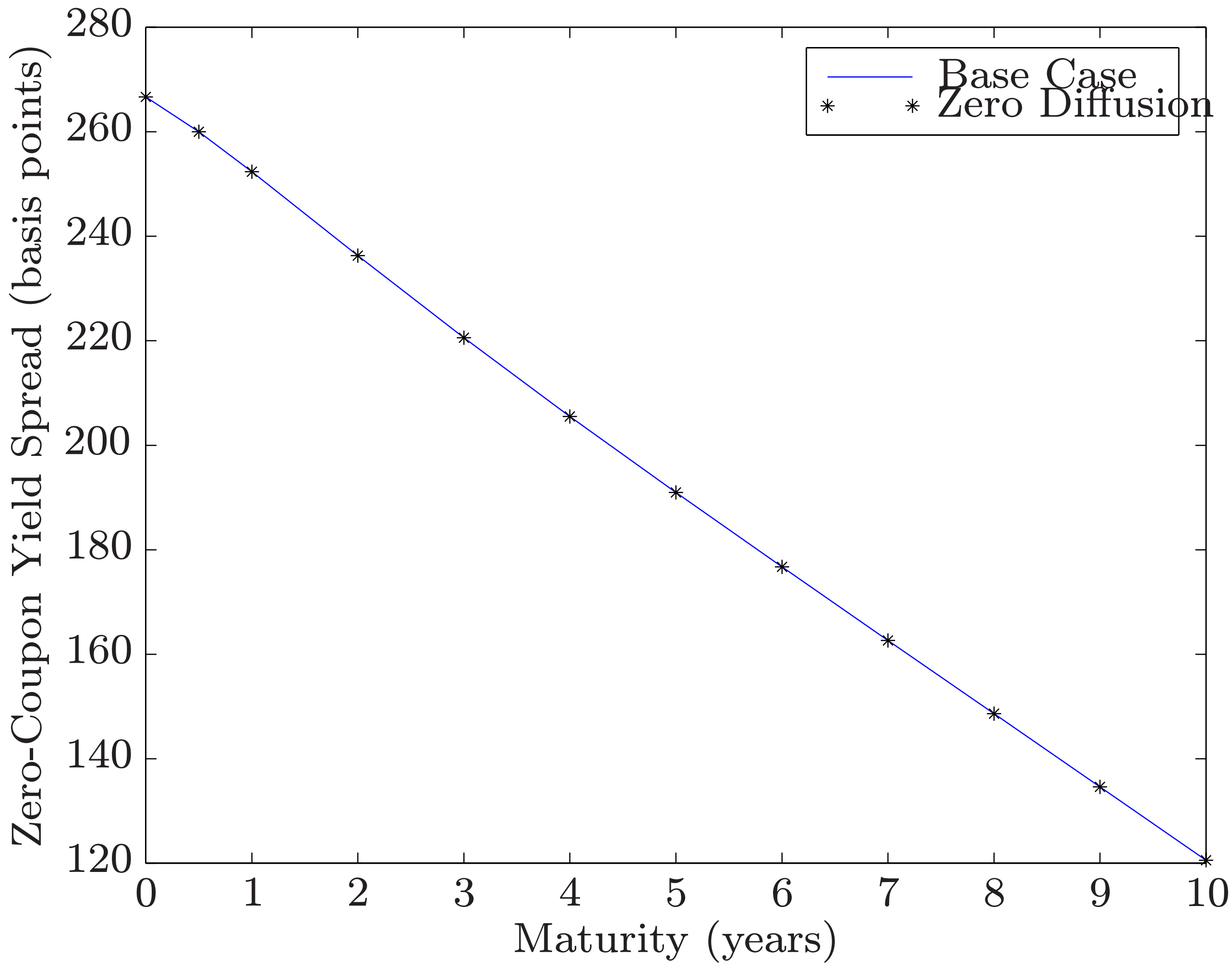


Figure 13: Zero-Coupon Yield Spreads, With and Without Diffusion

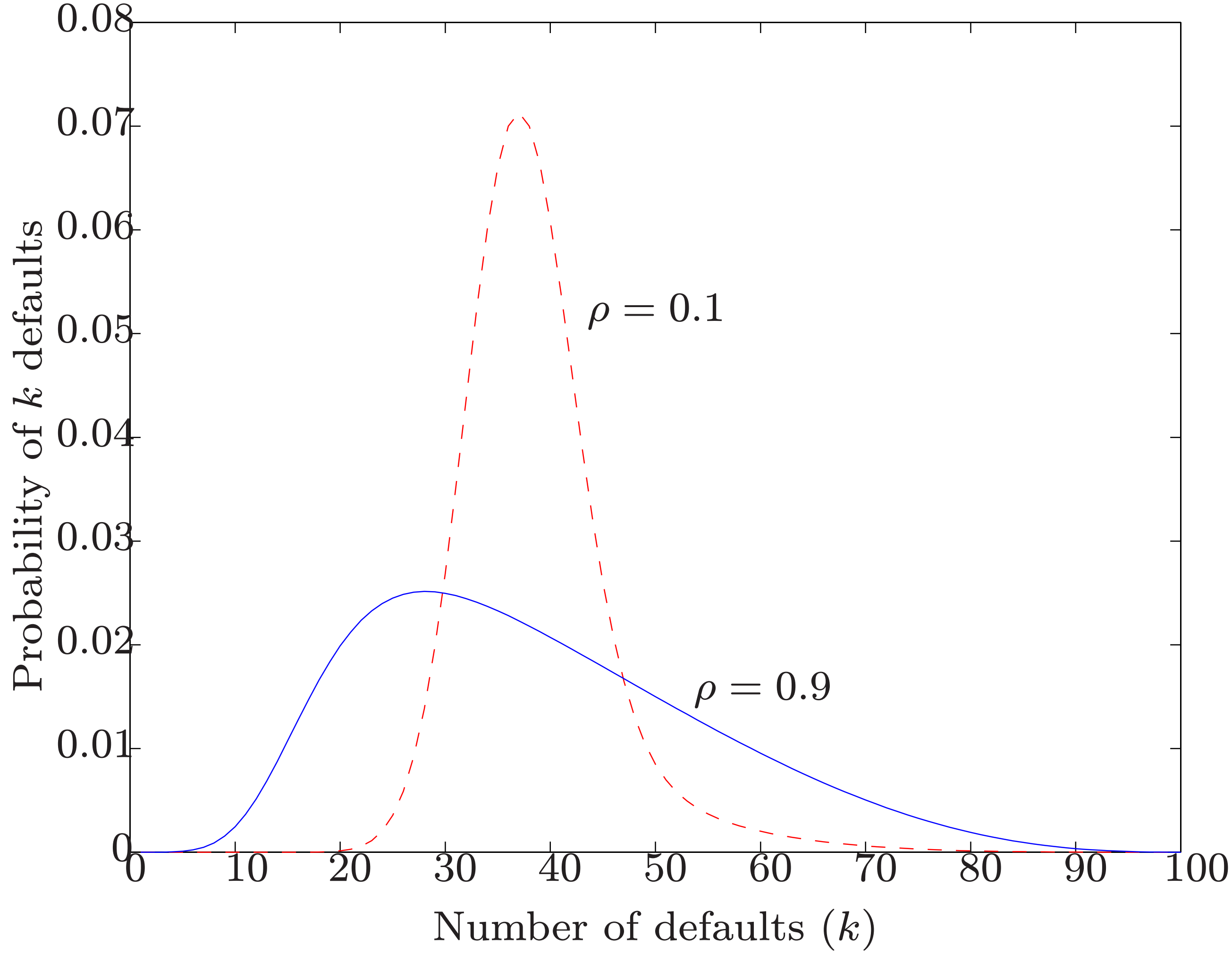


Figure 14: The probability of k defaults, for high and low correlation (base case).

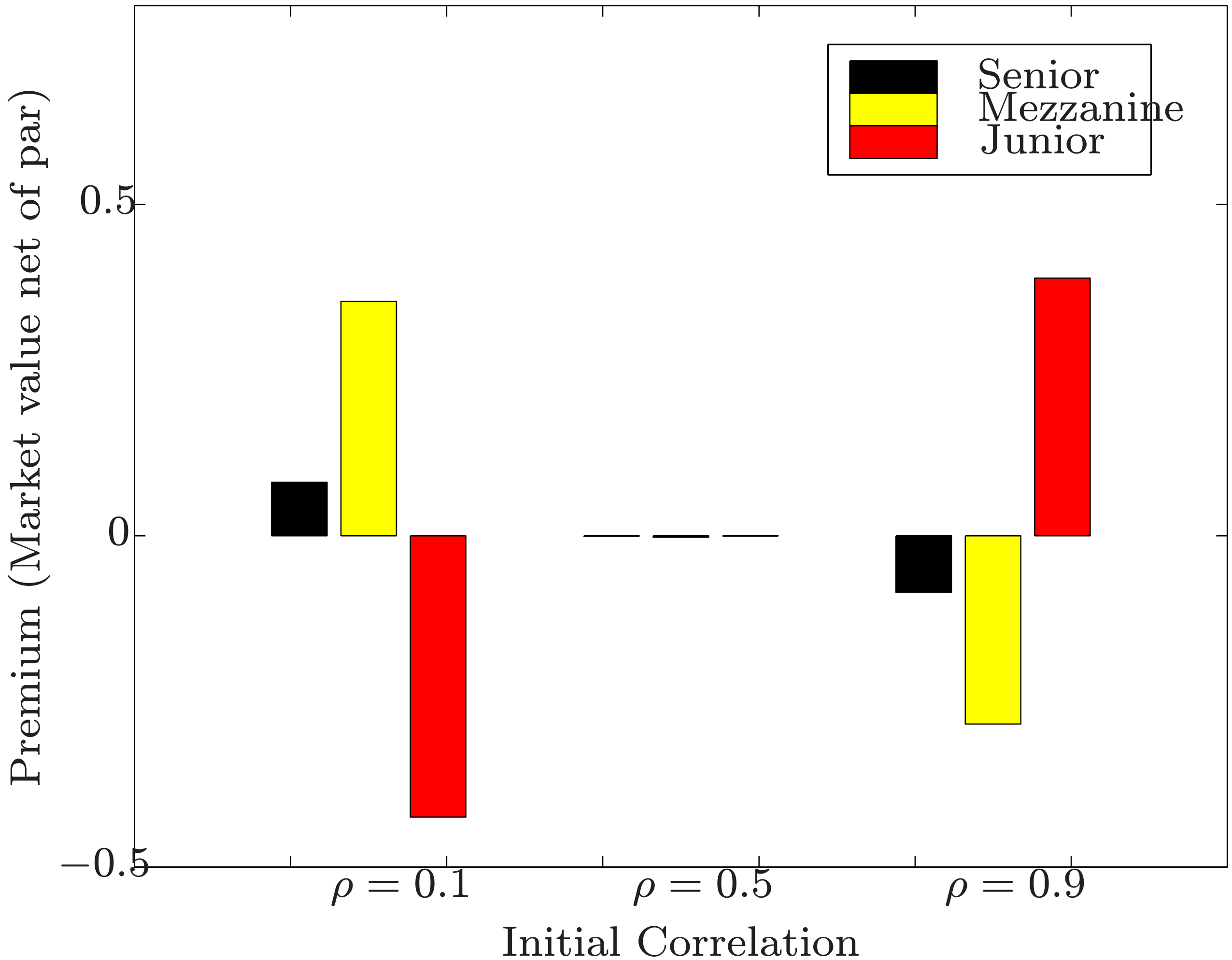


Figure 15: Impact on market values of correlation, uniform prioritization, Parameter Set 1, high overcollateralization ($P_1 = 80$).

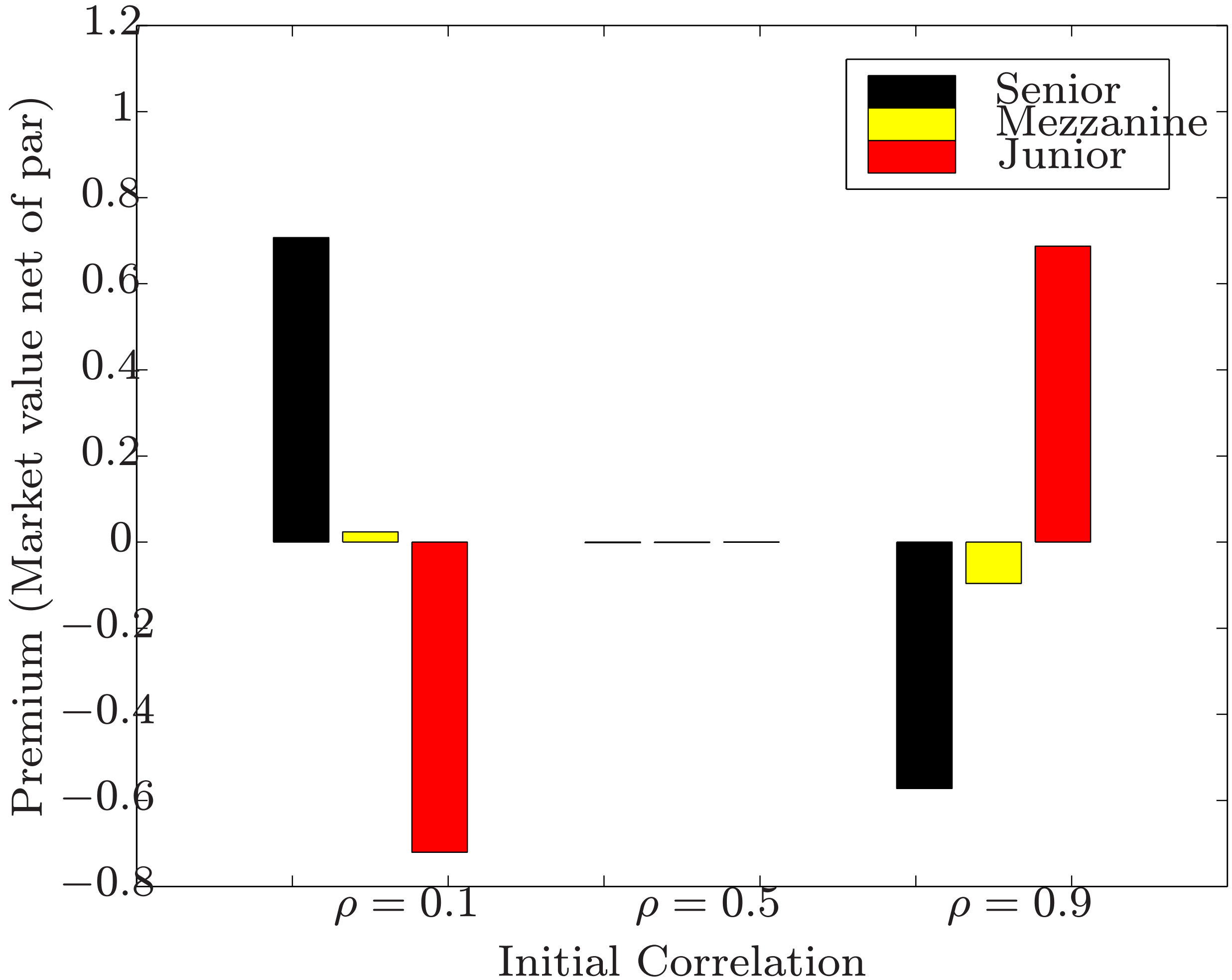


Figure 16: Impact on market values of correlation, uniform prioritization, Parameter Set 1, low overcollateralization ($P_1 = 92.5$).

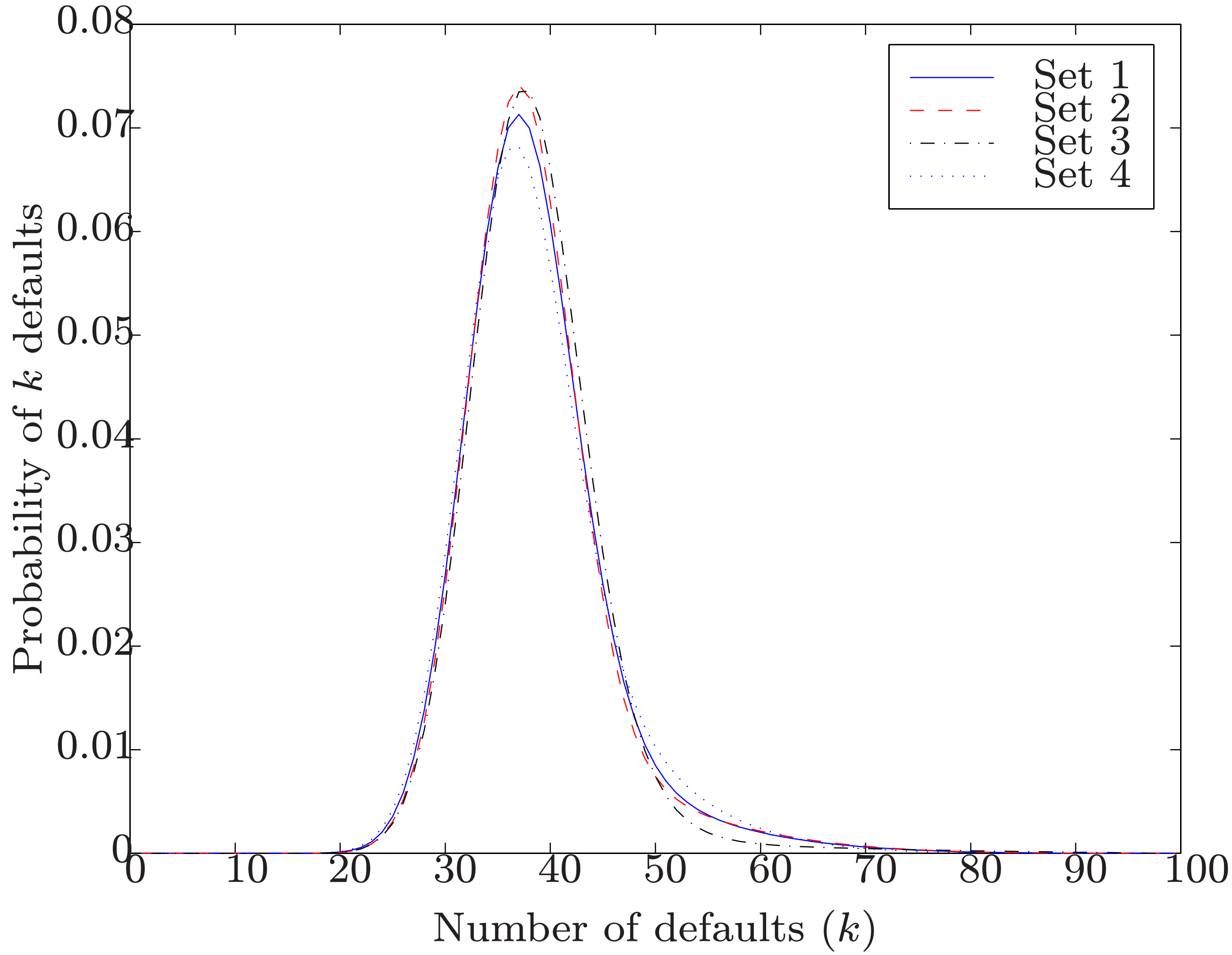


Figure 17: Probability of k defaults, all parameter sets, low correlation ($\rho = 0.1$).

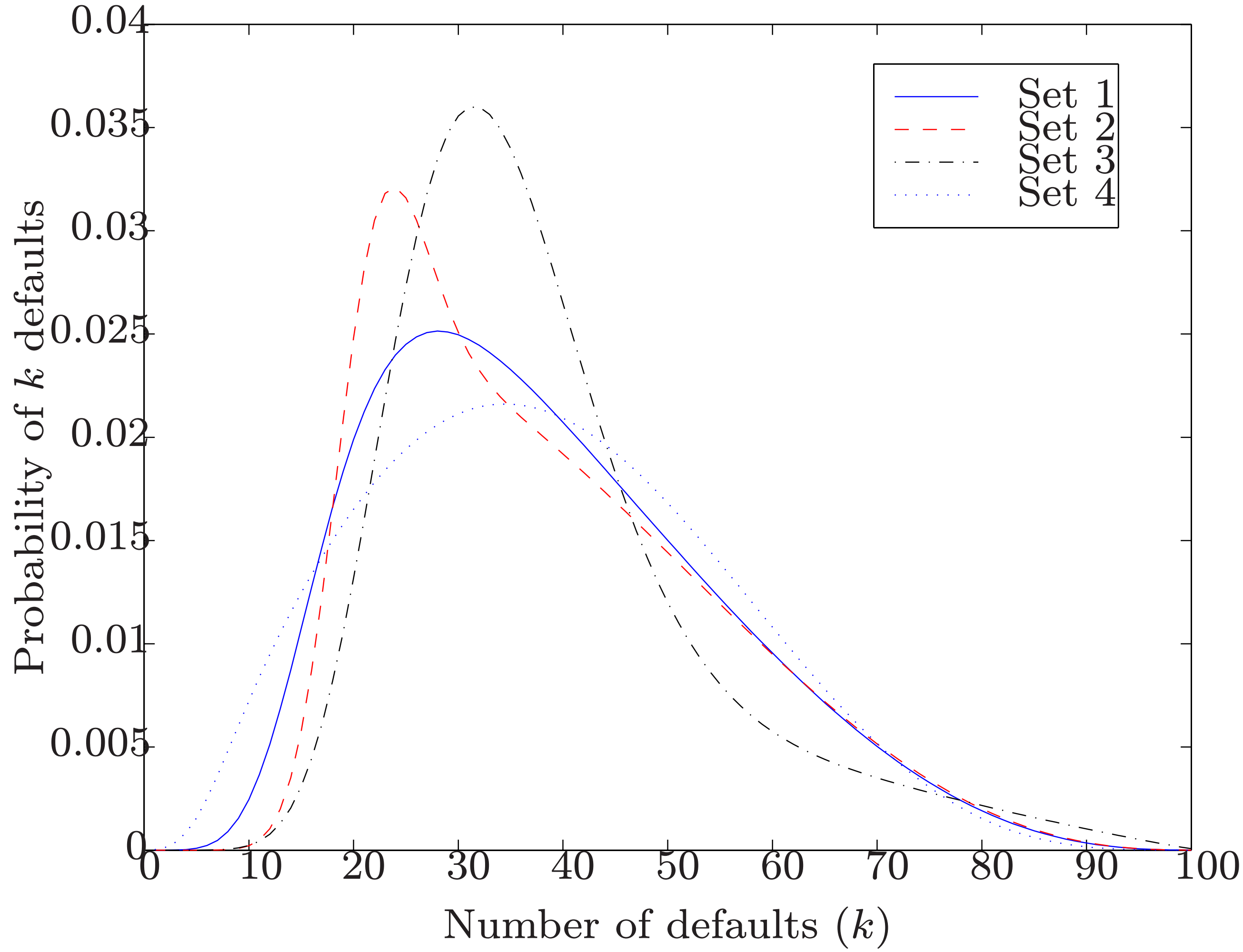


Figure 18: Probability of k defaults, all parameter sets, high correlation ($\rho = 0.9$).

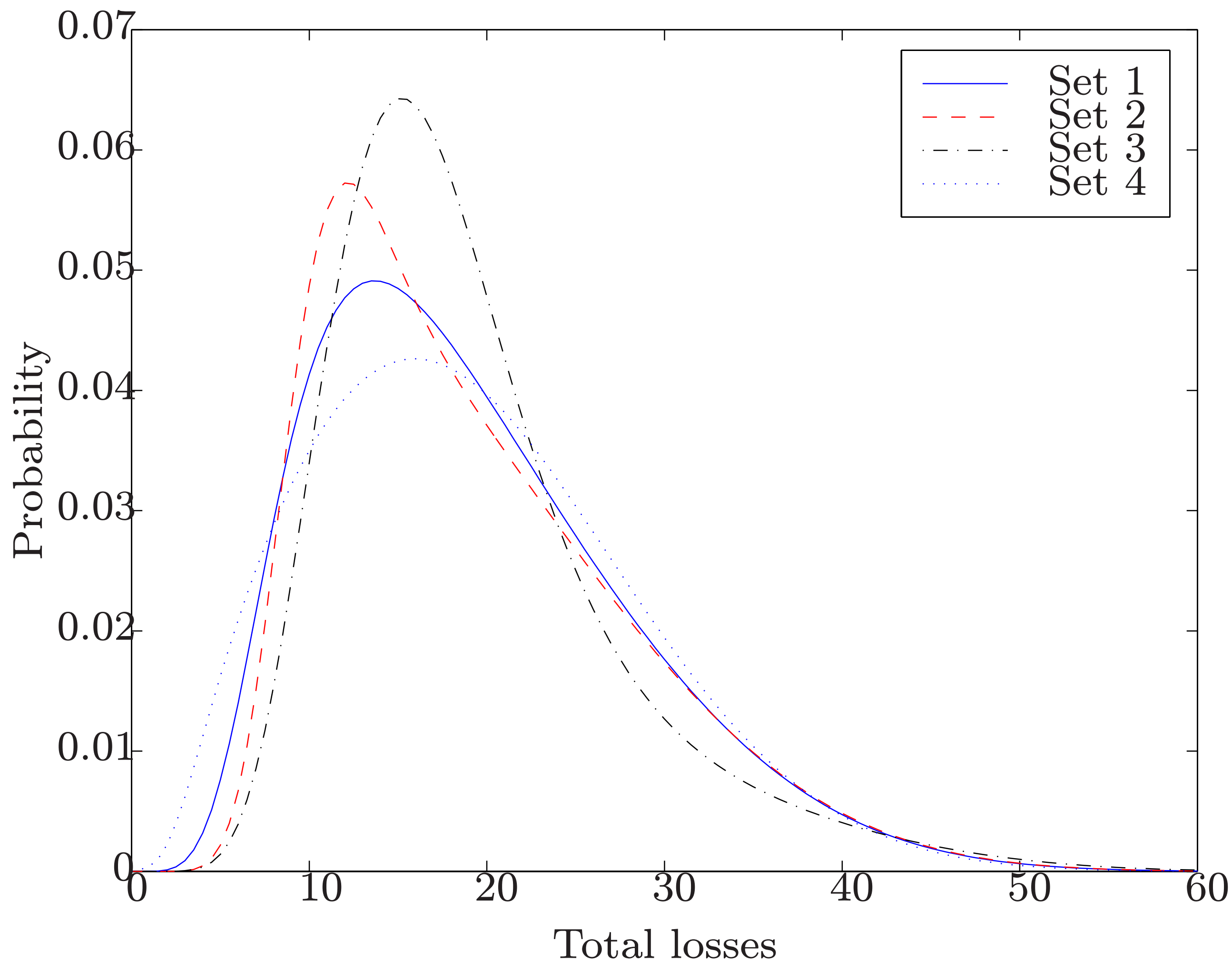


Figure 19: Probability Density of total losses of principal through default. (Parameter Set 1), high correlation.

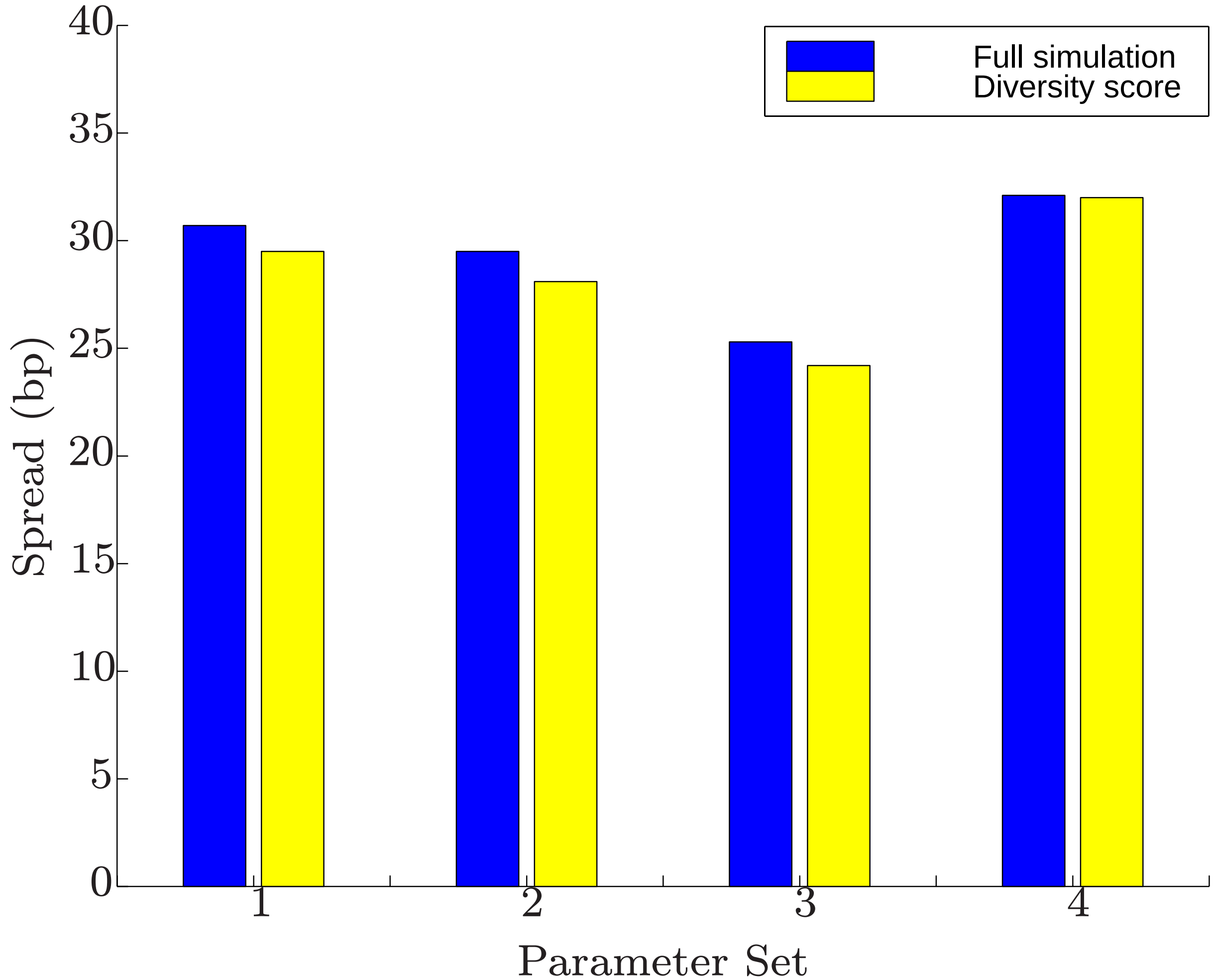


Figure 20: Senior Tranche spreads and diversity-based approximate spreads, high correlation ($\rho = 0.9$), low subordination ($P_1 = 92.5$).

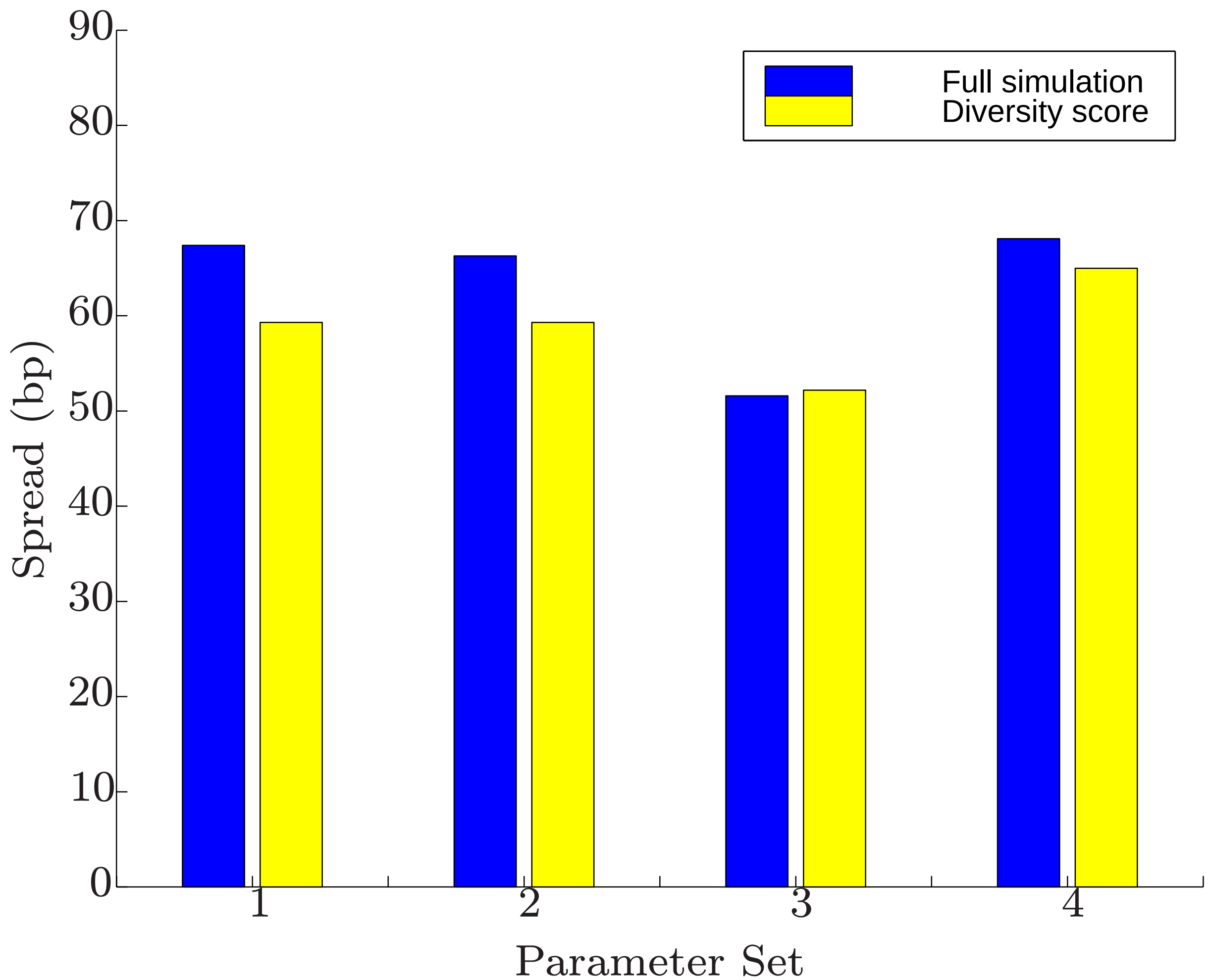


Figure 21: Mezzanine spreads and diversity-based approximate spreads, correlation $\rho = 0.5$, subordination: $(P_1 = 80,\ P_2 = 10)$.

Table 6: Par spreads ($\rho = 0.5$).

	Principal		Spread (Uniform)		Spread (Fast)	
Set	P_1	P_2	s_1 (bp)	s_2 (bp)	s_1 (bp)	s_2 (bp)
1	92.5	5	18.7 (1)	636 (16)	13.5 (0.4)	292 (1.6)
2	92.5	5	17.9 (1)	589 (15)	13.5 (0.5)	270 (1.6)
3	92.5	5	15.3 (1)	574 (14)	11.2 (0.5)	220 (1.5)
4	92.5	5	19.1 (1)	681 (17)	12.7 (0.4)	329 (1.6)
1	80	10	1.64 (0.1)	67.4 (2.2)	0.92 (0.1)	38.9 (0.6)
2	80	10	1.69 (0.1)	66.3 (2.2)	0.94 (0.1)	39.5 (0.6)
3	80	10	2.08 (0.2)	51.6 (2.0)	1.70 (0.2)	32.4 (0.6)
4	80	10	1.15 (0.1)	68.1 (2.0)	1.70 (0.2)	32.4 (0.6)