Barrier Option Pricing Using Adjusted Transition Probabilities

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Rotterdam, 25th Erasmus Finance day
May 11th, 2007

Outline

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1. Motivation
   - The standard Cox-Ross-Rubinstein Binomial Tree
   - Previous work
2. New Approach
   - Adjusted Binomial Tree
3. Results
   - Comparison
   - What is new?

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Convergence behaviour of the Standard Tree

- The convergence of the standard Cox-Ross-Rubinstein Binomial Tree displays an erratic behaviour and a persistent bias in pricing Barrier Options
- The reason for this behaviour, as pointed out by Boyle (1994), is that "the barrier will in general lie in between two adjacent nodes in the lattice"
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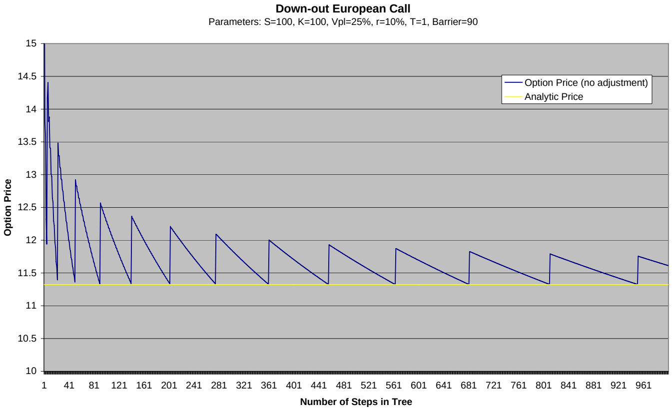
This Is the Problem



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Boyle and Lau (1994) (1)

Revised Binomial Tree

- In case of a constant barrier (H), it is easy to constrain the time partition such that the barrier lies just above a layer of horizontal nodes
- Recall that, in the CRR model, the down (up) movement is equal to $d = \exp\left(-\sigma\sqrt{T/n}\right)$ $\left(u = \exp\left(\sigma\sqrt{T/n}\right)\right)$
- In order to obtain the desired result, it is enough to select n (the number of steps in the tree) such that it is the largest integer smaller than

$$F(m) = \frac{m^2\sigma^2 T}{\left(\ln S/H\right)^2} \quad m = 1, 2, \cdots \tag{1}$$

where m is the number of down (up) steps that takes the asset price just above (below) the barrier

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"Stretched" Trinomial Tree

- Ritchken proposes an extension of the standard trinomial model
- As usual, the issue is to place the nodes as near as possible (from above) to the barrier
- Recall that, in the trinomial model, *up* and *down* movements are respectively $up = \exp\left(\lambda\sigma\sqrt{T/n}\right)$ and $down = \exp\left(-\lambda\sigma\sqrt{T/n}\right)$
- It is possible to select the stretch parameter λ such that the barrier lies exactly on the nodes
- For more complex options, such as, options with double constant barrier, Ritchken proposes a state dependent tree with two stretch factors that allow to reposition the tree on the barriers

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- If a parameter of the option changes (maturity, volatility, etc...) then the entire lattice must be repositioned before calculating the new option price
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Further Extensions

- Costabile (2001) proposes a discrete time algorithm for pricing double constant barrier options
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The problem with these procedures is that they are “ad hoc” solutions to specific cases; we look for a general algorithm that allows to approximate simple (single constant barrier) as well as complex (single and double time-varying) barriers options.

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Probability Adjustment

- Baldi, Caramellino and Iovino (1999) derive a series of approximations for the exit probability of the Brownian bridge that can be used to price multiple and time-varying barriers options
- Recall that the log of the asset price follows a Brownian Motion
- Baldi et al. use these probabilities to improve the Monte Carlo calculations, because with the standard procedure it is possible that the underlying asset price breaches the barrier without being detected
- Here we use these probabilities to improve the Cox-Ross-Rubinstein binomial tree

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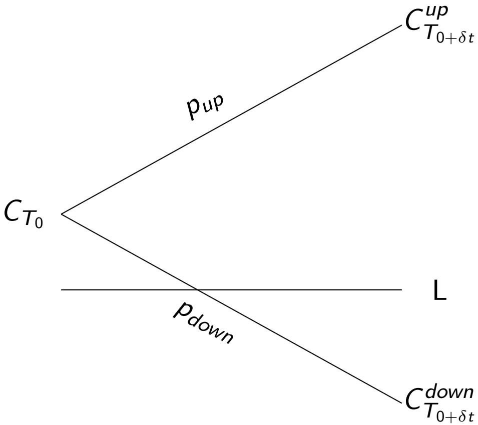
Procedure

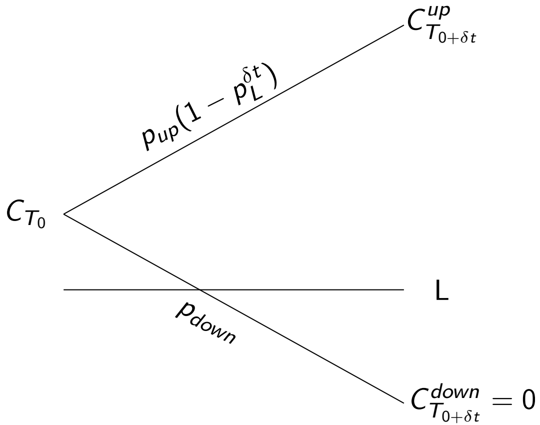
A Simple Example: Down-out call (DOC) with constant barrier (1)



The "adjusted tree" is similar to the standard CRR model; the difference relies on the computation of the value of the option in the nodes just above the barrier.

Procedure

A Simple Example: Down-out call (DOC) with constant barrier (2)



That is, we multiply the usual probability of reaching the up point by one minus the conditional probability $(p_L^{\delta t})$ that the assets price, in the time interval, hits the barrier.

Procedure

A Simple Example: Down-out call (DOC) with constant barrier (3)

- $p_L^{\delta t}$ = Exit Probability. This is the probability that the asset price, starting from S_{T_0} and ending in $S^{up}_{T_{0+\delta t}}$, hits the barrier
- In this case we have

$$p_L^{\delta t}(T_0, S_{T_0}, S^{up}_{T_{0+\delta t}}, L) = \exp\left(-\frac{2}{\sigma^2 \delta t}\right) \ln\left(\frac{S_{T_0}}{L}\right) \ln\left(\frac{S^{up}_{T_{0+\delta t}}}{L}\right)$$

- With this probability adjustment the price of the DOC near the barrier (when $S^{down}_{T_{0+\delta t}} < L$), at time T_0, is

$$C_{DOC} = \exp(-r\delta t) p_{up} (1 - p_L^{\delta t}) C(S^{up}_{T_{0+\delta t}})$$

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- With this probability adjustment the price of the DOC near the barrier (when $S_{T_{0+\delta t}}^{down} < L$), at time T_0, is

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Procedure

General Case

- The same procedure can be extended to the pricing of more complex options with single and multiple time-varying barriers.
- Baldi at al. provide exit probability approximations for all kinds of time-varying barriers
- In the paper we focus on:
  - Single/double constant barriers
  - Single/double exponential barriers
  - Single/double (time) linear barriers
- This simple procedure avoids the need of repositioning the tree "on the barrier"

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Down-out European Call Price Approximation (1)

Number of Time Steps in the Tree							
Stk. P.	500	1000	2000	3000	4000	5000	Alyt. P.
94.0	4.910 (4.863)	4.957 (4.864)	4.915 (4.864)	4.920 (4.864)	4.852 (4.864)	4.886 (4.864)	4.864
93.0	3.720 (3.70)	3.716 (3.701)	3.733 (3.702)	3.715 (3.701)	3.728 (3.701)	3.722 (3.702)	3.702
92.0	2.500 (2.504)	2.589 (2.506)	2.515 (2.506)	2.546 (2.506)	2.563 (2.506)	2.521 (2.506)	2.506
91.5	2.047 (1.894)	1.901 (1.894)	1.894 (1.895)	1.963 (1.895)	1.907 (1.895)	1.945 (1.895)	1.895
91.0	1.242 —	1.365 (1.274)	1.263 (1.274)	1.331 (1.275)	1.315 (1.275)	1.279 (1.274)	1.274
90.5	0.810 —	0.758 —	0.624 —	0.663 —	0.691 (0.642)	0.699 (0.642)	0.642

Table: K=100, Vol=25%, r=10%, T=1, L=90, ()=Ritchken'96.

Down-out European Call Price Approximation (2)

Number of Time Steps in the Tree							
Stk. P.	500	1000	2000	3000	4000	5000	Alyt. P.
90.4	0.649 —	0.642 —	0.576 —	0.508 —	0.521 —	0.537 (0.515)	0.515
90.3	0.476 —	0.490 —	0.479 —	0.450 —	0.419 —	0.390 —	0.387 —
90.2	0.303 —	0.316 —	0.327 —	0.328 —	0.323 —	0.316 —	0.258
90.1	0.142 —	0.146 —	0.152 —	0.156 —	0.159 —	0.161 —	0.129 —
90.05	0.068 —	0.069 —	0.071 —	0.072 —	0.073 —	0.074 —	0.065
90.01	0.013 —	0.013 —	0.013 —	0.013 —	0.013 —	0.013 —	0.013

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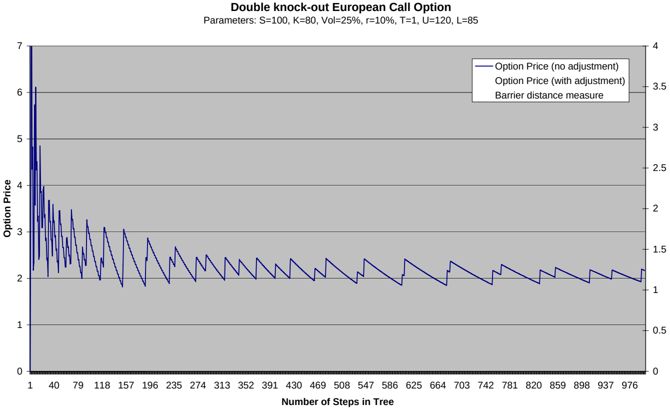
Convergence Behaviour

Double Knock-out European Call with Constant Barriers



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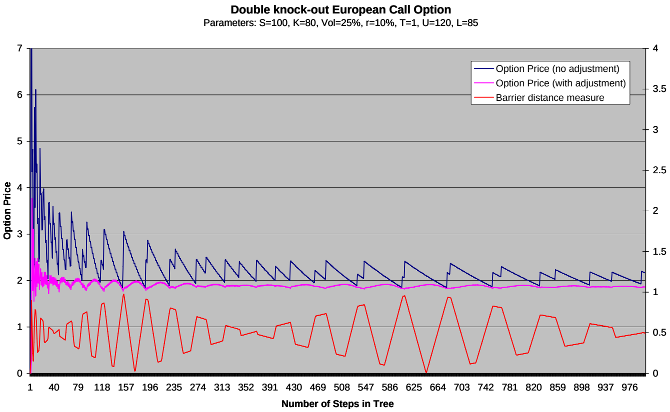
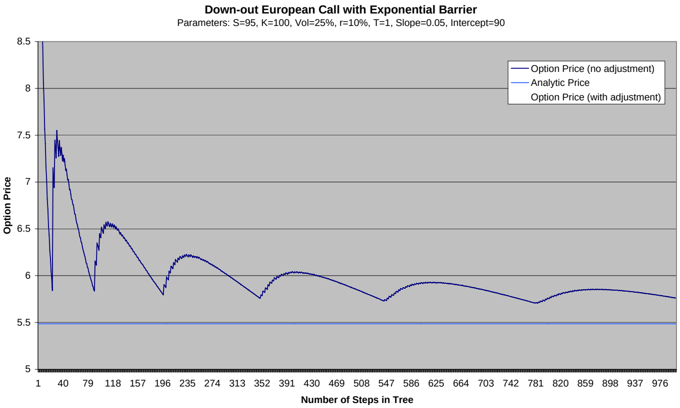


Convergence Behaviour

Down-out European Call with Exponential Barrier



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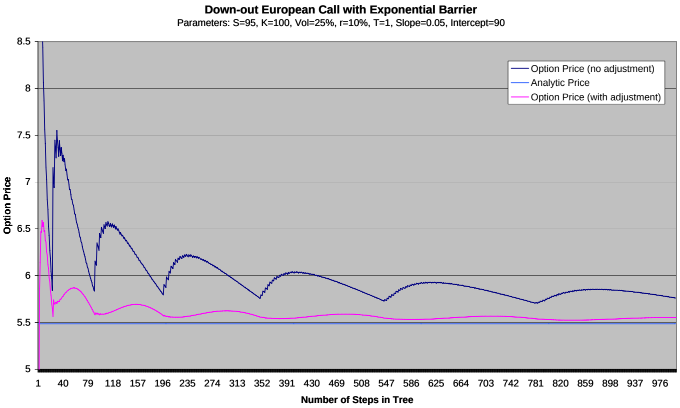

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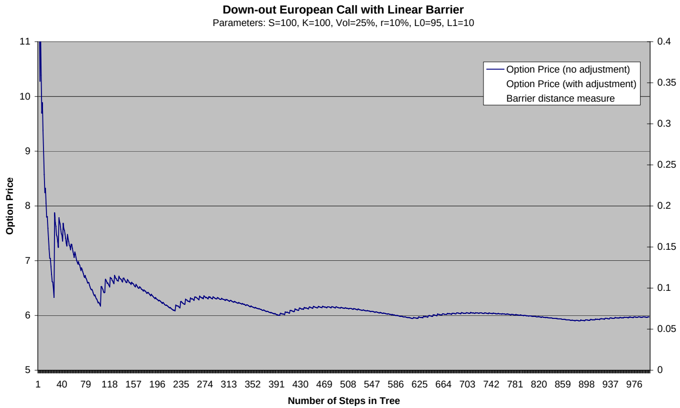

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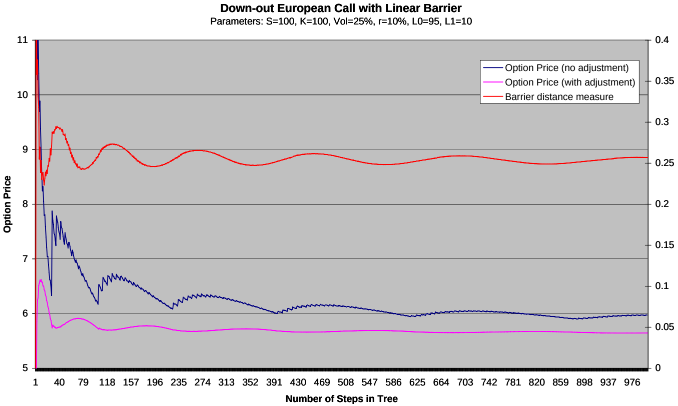

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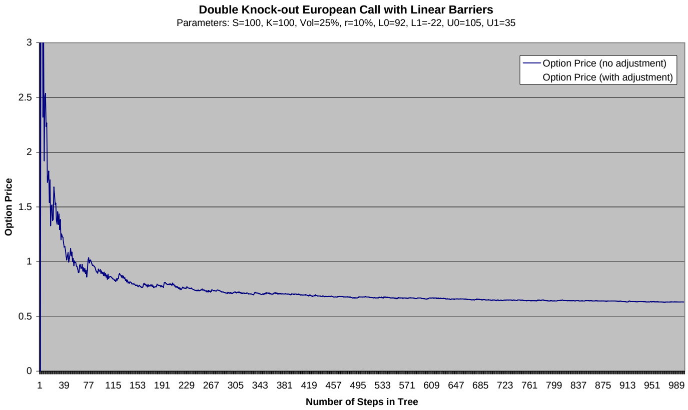

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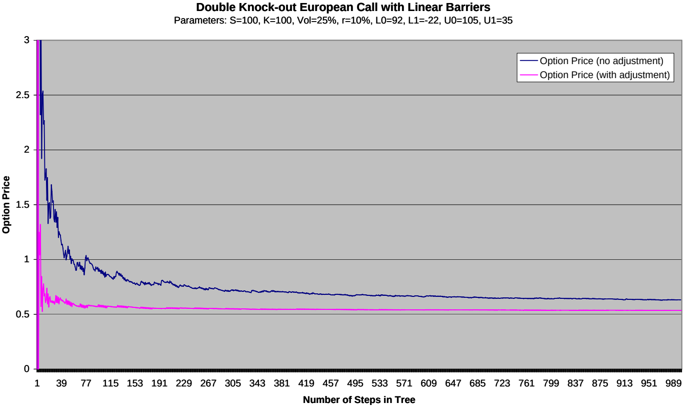

Double Knock-out European Call with Short Maturity

Vol	U	L	KI	FD	Approx1	Approx2
$\sigma = 0.20$	1500	500	25.12	24.47	25.12	25.12
	1200	800	24.16	24.69	24.77	24.76
	1050	950	2.15	2.15	2.18	2.17
$\sigma = 0.30$	1500	500	36.58	36.04	36.59	36.58
	1200	800	29.45	29.40	29.52	29.46
	1050	950	0.27	0.27	0.28	0.28
$\sigma = 0.40$	1500	500	47.58	47.31	47.86	47.85
	1200	800	25.84	25.82	25.89	25.94
	1050	950	0.02	0.01	0.02	0.02

Table: U = Upper Barrier, L = Lower Barrier, KI = Kunitomo and Ikeda method, FD = Finite Difference method, Approx1 = 1000 time-divisions, Approx2 = 2000 time-divisions. Parameters: $S_0 = 1000, K = 1000, r = 5\%, T = 0.5$.

Down-out European Call with Exponential Barrier

	Slope=-0.1			Slope=0.1	
Tree Lvls.	Cost.	Adj.	Tree Lvls	Cost.	Adj.
17	7.002	6.841	24	5.020	5.227
77	6.958	6.871	92	4.949	5.091
181	6.920	6.920	203	4.934	5.041
327	6.910	6.930	356	4.934	5.016
515	6.912	6.935	552	4.932	4.999
2100	6.902	6.927	2174	4.929	4.964
4754	6.900	6.919	4865	4.929	4.952
Analytic	6.896		Analytic	4.928	

Table: Comparison of results between the adjusted-probability method and the extended Cox-Ross-Rubinstein method of Costabile(2002)

Double Knock-out European Call

Tree Lvls.	Option Value Case 1	Option Value Case 2	Option Value Case 3
1000	0.0414	0.0184	0.0774
2000	0.0412	0.0181	0.0765
3000	0.0412	0.0182	0.0767
4000	0.0413	0.0181	0.0765
5000	0.0411	0.0181	0.0765
Analyt. Value	0.0411	0.0178	0.0762

Table: Comparison of results between the adjusted-probability method and the analytical values calculated by Geman and Yor (1996).
Case1: $S_0 = 2, K = 2, \sigma = 20\%, r = 2\%, T = 1y, L = 1.5, U = 2.5$
Case2: $S_0 = 2, K = 2, \sigma = 50\%, r = 5\%, T = 1y, L = 1.5, U = 3$
Case3: $S_0 = 2, K = 1.75, \sigma = 50\%, r = 5\%, T = 1y, L = 1.5, U = 3$

Outline

Goals of the Paper

The "Adjusted Tree":

- does not require to reposition the tree "on the barrier"
- demonstrates good convergence properties towards the analytical price for both single and double constant barriers options
- produces an accurate approximation when the stock price is very close to the barrier
- produces an accurate approximation for options with short-term maturity
- can produce price approximation for time-varying barrier options including exponential, single linear and double linear barriers

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- does not require to reposition the tree "on the barrier"
- demonstrates good convergence properties towards the analytical price for both single and double constant barriers options
- produces an accurate approximation when the stock price is very close to the barrier
- produces an accurate approximation for options with short-term maturity
- can produce price approximation for time-varying barrier options including exponential, single linear and double linear barriers

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