Workshop and Mid-Term Conference on Advanced Mathematical Methods for Finance September, 17th-22nd, 2007

Advanced Mathematical Methods for Finance

Monday, September 17th

Dorje C. Brody - "Information-Based Asset Pricing":
A new framework for asset price dynamics (due to Brody, Hughston, and Macrina) is introduced in which the concept of noisy information about future cash flows is used to derive the corresponding price processes. In this framework an asset is defined by its cash-flow structure. Each cash flow is modelled by a random variable that can be expressed as a function of a collection of independent random variables called market factors. with each such "X-factor" we associate a market information process, the values of which are accessible to market participants. Each information process consists of a sum of two terms; one contains true information about the value of the associated market factor, and the other represents "noise". The noise term is modelled by an independent Brownian bridge that spans the interval from the present to the time at which the value of the factor is revealed. The market filtration is generated by the aggregate of the independent information processes. The price of an asset is given by the expectation of the discounted cash flows in the risk-neutral measure, conditional on the information provided by the market filtration. In the case where the cash flow is the payment of a defaultable bond, an explicit model is obtained for the price process of a credit-risky bond, for which the associated derivative price and hedging strategies are also obtained. In the case where the cash flows are the dividend payments associated with equities, an explicit model is obtained for the share-price process. The prices of options on dividend-paying assets are derived. Remarkably, the resulting formula for the price of a European-style call option is of the Black-Scholes type. The information-based framework allows for a reasonable way to accommodate dependencies across different assets, and also generates a natural explanation for the origin of stochastic volatility in financial markets, without the need for specifying on an ad hoc basis the dynamics of the volatility.

Ernst Eberlein - "Lévy Driven Equity, FX- and Interest Rate Models":
Empirical analysis of data from the financial markets reveals that standard diffusion models do not generate return distributions with a sufficient degree of accuracy. to reduce model risk we study models which are driven by Lévy processes or more general by semimartingales. Analytical properties of this model class are investigated. For implementation in particular the class of generalized hyperbolic Lévy processes is considered. Plain vanilla as well as exotic options are priced on the basis of these models. As a further application in risk management we show that estimates of the value at risk of a portfolio of securities are improved.
In the second part we discuss Lévy term structure models. Three basic approaches to model interest rates are introduced: the forward rate model, the forward process model, and the LIBOR or market model. As an application pricing formulae for caps and floors are derived. Efficient algorithms to evaluate these formulae numerically are given. The LIBOR model can be extended to a multi-currency setting. Closed form pricing formulae for cross-currency derivatives such as foreign caps and floors and cross-currency swaps are studied in detail. The LIBOR model can also be extended to include defaultable instruments.

Michèle Vanmaele- "Comonotonicity Applied in Finance":
In finance very often one has to deal with problems where multivariate random variables are involved, e.g. basket options where the price depends on several underlying securities. Asian options or Asian basket options are other examples of options where the price depends on a weighted sum of non-independent asset prices.
One can construct upper and lower bounds for the prices of such types of European call and put options based on the theory of stochastic orders and of comonotonic risks. Comonotonicity essentially reduces a multivariate problem to a univariate one, leaving the marginal distributions intact.
One can model the dynamics of the underlying asset prices, e.g. use a Black-Scholes model; but it is also possible to develop model-free bounds expressed in terms of in the market observed option prices on the individual underlying assets. Moreover the comonotonic upper bound can be interpreted as a superreplicating strategy.

Instead of deriving bounds one can look at approximations, e.g. Monte Carlo (MC) simulation is a technique that provides approximate solutions to a broad range of mathematical problems. A drawback of the method is its high computational cost, especially in a high-dimensional setting. Therefore variance reduction techniques were developed to increase the precision and reduce the computer time. the so-called Comonotonic Monte Carlo simulation uses the comonotonic approximation as a control variate to get more accurate estimates and hence a less time-consuming simulation.
We will introduce the notion of comonotonicity and discuss the different approaches based on the theory of comonotonicity. The methods will be applied to examples in finance.