

# Lévy driven equity, FX- and interest rate models

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## Abstract

Empirical analysis of data from the financial markets reveals that standard diffusion models

$$dS_t = \mu(t, S_t)dt + \sigma(t, S_t)dW_t$$

do not generate return distributions for stock prices, indices, or exchange rates with a sufficient degree of accuracy. To reduce model risk we introduce models of the form

$$S_t = S_0 \exp(X_t)$$

which are driven by Lévy processes or more general by semimartingales  $(X_t)_{t \geq 0}$  instead of a Brownian motion  $(W_t)_{t \geq 0}$ . Analytical properties of this model class are investigated. For implementation in particular the class of generalized hyperbolic Lévy processes is considered. Plain vanilla as well as exotic options are priced on the basis of this model. As a further application in risk management we show that estimates of the value at risk of a portfolio of securities are improved.

In the second part we discuss Lévy term structure models. Three basic approaches to model interest rates are introduced: the forward rate model, the forward process model, and the LIBOR or market model. As an application pricing formulae for caps and floors are derived. Efficient algorithms to evaluate these formulae numerically are given. The LIBOR model can be extended to a multi-currency setting. Closed form pricing formulae for cross-currency derivatives such as foreign caps and floors and cross-currency swaps are studied in detail. The LIBOR model can also be extended to include defaultable instruments.

## References

- [1] E. Eberlein: Application of generalized hyperbolic Lévy motions to finance. In: *Lévy Processes: Theory and Applications*, O.E. Barndorff-Nielsen, T. Mikosch, and S. Resnick (eds.), Birkhäuser Verlag (2001) 319–337
- [2] E. Eberlein, E. A. von Hammerstein: Generalized hyperbolic and inverse Gaussian distributions: limiting cases and approximation of processes. In: *Seminar on Stochastic Analysis, Random Fields and Applications IV*, Progress in Probability 58, R.C. Dalang, M. Dozzi, F. Russo (eds.), Birkhäuser Verlag (2004) 221–264
- [3] E. Eberlein, J. Jacod, S. Raible: Lévy term structure models: no-arbitrage and completeness. *Finance and Stochastics* **9** (2005) 67–88
- [4] E. Eberlein, U. Keller: Hyperbolic distributions in finance. *Bernoulli* **1** (1995) 281–299
- [5] E. Eberlein, U. Keller, K. Prause: New insights into smile, mispricing and value at risk: the hyperbolic model. *Journal of Business* **71** (1998) 371–405
- [6] E. Eberlein, W. Kluge: Exact pricing formulae for caps and swaptions in a Lévy term structure model. *Journal of Computational Finance* **9** (2) (2006) 99–125
- [7] E. Eberlein, W. Kluge, P. J. Schönbucher: The Lévy Libor model with default risk. *Journal of Credit Risk* **2** (2) (2006) 3–42
- [8] E. Eberlein, N. Koval: A cross-currency Lévy market model. *Quantitative Finance* **6** (2006) 465–480
- [9] E. Eberlein, F. Özkan: The Lévy Libor Model. *Finance and Stochastics* **9** (2005) 327–348
- [10] E. Eberlein, K. Prause: The generalized hyperbolic model: financial derivatives and risk measures. In: *Mathematical Finance—Bachelier Congress 2000*, H. Geman, D. Madan, S. Pliska, T. Vorst (eds.), Springer Verlag (2002), 245–267
- [11] E. Eberlein, S. Raible: Term structure models driven by general Lévy processes. *Mathematical Finance* **9** (1999) 31–54